Inherently Balanced Double Bennett Linkage

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Abstract. For fast moving mechanisms shaking force balance is important to reduce base vibrations. When mechanisms are force balanced, they are also gravity balanced which is important for reduced actuation effort and increased safety. It has been shown that from planar inherently balanced linkage architectures a variety of new and interesting force balanced mechanism solutions can be synthesized. The goal of this paper is to derive the balance conditions of one special solution consisting of two similar 4R four-bar linkages connected by a parallelogram, based on two sets of principal vectors. It is shown that also here the balance conditions can be derived from the linear momentum equations of each relative degree of freedom. Subsequently it is shown how the planar version can be transformed into a spatial version becoming an inherently balanced linkage of two similar Bennett linkages connected by a planar parallelogram. The balance conditions for both the planar and spatial version are exactly equal.

Key words: Inherent force balance, Bennett linkage architecture, mass motion, principal vectors

1 Introduction

When mechanisms (i.e. robotic manipulators) move at high-speeds, base vibrations due to the dynamic reactions on the base (the so called shaking forces and shaking moments) are generally significant. These vibrations limit the performance of mechanisms which cannot run as fast and precise as desired. Contrary to common solutions to minimize the influence of base vibrations such as damping and advanced control, it is also possible to design a mechanism such that it does not produce any base vibrations at all. The mechanism then is designed dynamically (shaking force and shaking moment) balanced [2, 3].

The main challenge in designing balanced mechanisms is to limit the increase of mass, inertia, and complexity of the design for an advantageous application [6]. Instead of balancing a pre-existing mechanism, an approach where dynamic balance forms the starting point of the design based on which suitable mechanism architectures are synthesized has shown to lead to a variety of new balanced mechanism. This approach is named inherent dynamic balancing [3].

The inherent dynamic balance approach is based on the method of principal vectors, describing the motion of link masses relative to the center of mass (CoM) of the complete linkage in a specific decoupled way. In [5] for the first time a 'grand' inherently balanced linkage architecture was presented, with the novelty that it is not based on solely one principal vector set, but on the combination of all possible principal vector sets. From this highly overconstrained but movable architecture it was shown that a variety of new and interesting balanced linkages could be found.

In this paper it is shown how the balance conditions can be derived of one of the inherently balanced mechanism solutions which is based on two principal vector sets. The mechanism consists of two similar four-bar linkages that are connected with a parallelogram. It is shown also how from this planar linkage a spatial inherently balanced double Bennett linkage is found.

First the planar linkage is explained and the force balance conditions are derived from the linear momentum equations of each DoF independently. Here one element in each closed chain is modeled mass equivalently. Subsequently it is shown how the spatial inherently balanced Bennett linkage architecture can be obtained from the planar version.

2 Planar inherently balanced linkage with two similar four-bars

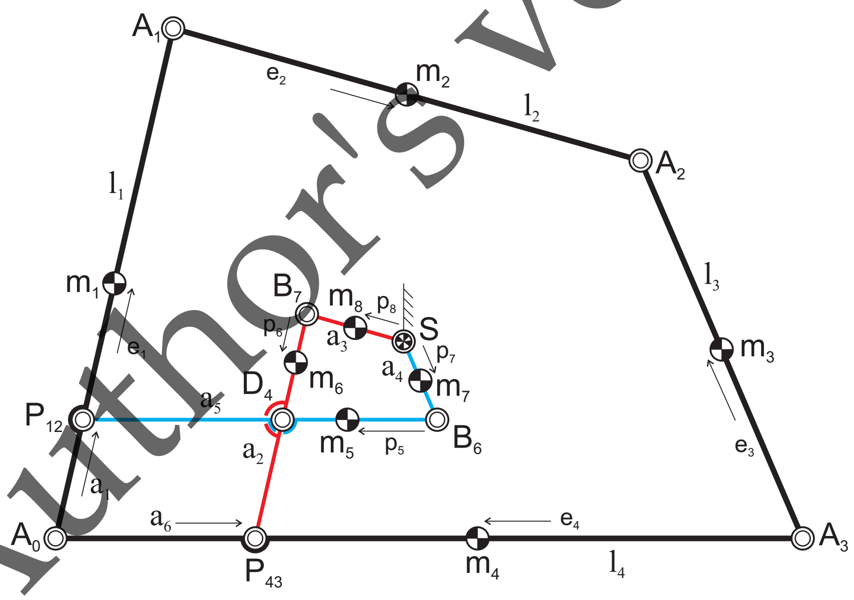

Fig. 1 Inherently balanced 2-DoF planar 4R four-bar based linkage with 8 links from [5], drawn to scale for a realistic impression. Four-bar $A_0A_1A_2A_3$ is similar to four-bar $D_4B_7SB_6$ and moves synchronously for all motion with the common CoM S stationary in the base pivot.

Figure 1 shows the basic inherently balanced linkage solution that was derived in [5] from the so called 'grand 4R four-bar based inherently balanced linkage architecture', a highly overconstrained but movable architecture including all principal vector links of the four different principal vector sets. The linkage is drawn to scale for a realistic impression and consists of eight links which are organized as two similar four-bar linkages $A_0A_1A_2A_3$ and $D_4B_7SB_6$ with solely revolute pairs. The four-bar linkages move synchronously for all motion which is induced by the parallelogram $A_0P_{12}D_4P_{43}$ of which $P_{12}D_4$ is part of link $P_{12}B_6$ and D_4P_{43} is part of link B_7P_{43}. The links of four-bar $A_0A_1A_2A_3$ have lengths l_i whereas the inner links have lengths a_2, a_3, a_4, and a_5. Joints P_{12} and P_{43}, which are two principal points, are located at a distance a_1 and a_6 from A_0, respectively as illustrated. The parameters a_i are principal dimensions of two different sets of principal vectors in the grand architecture. The conditions for similarity of the two four-bars can be described as:

$$\frac{l_1}{a_2-a_1}=\frac{l_2}{a_3}=\frac{l_3}{a_4}=\frac{l_4}{a_5-a_6} \tag{1}$$

with $a_2-a_1=|B_7D_4|$ and $a_5-a_6=|B_6D_4|=h_4$. Each link i has a mass m_i of which the CoM is located along the lines through the joints (i.e. all links are assumed mass symmetric with respect to these lines) defined by parameters e_i for the outer four-bar and by p_i for the inner four-bar as illustrated. The common CoM of all links together is in joint S for all motion of the linkage. By choosing joint S as the pivot with the base, the common CoM is stationary for which for all motion the linkage is shaking force balanced and gravity balanced. About S the linkage has two-degree-of-freedom (2-DoF) motion.

Also for a mechanism based on multiple sets of principal vectors such as the mechanism under investigation, the force balance conditions or conditions for which the common CoM is in joint S for all motion can be derived from the linear momentum equations of each relative DoF individually as explained in [3, 4]. Then first one element in each closed loop is modeled mass equivalently as shown in Fig. 2 to obtain an open-loop mass equivalent linkage. The mass m_2 of link A_1A_2 is modeled with equivalent masses m_2^a and m_2^b in joints A_1 and A_2, respectively, with the conditions for mass equivalence $m_2^a+m_2^b=m_2$ and $m_2^ae_2=m_2^b(l_2-e_2)$. Similarly for link SB_7 the mass m_8 can be modeled with equivalent masses m_8^a and m_8^b in joints S and B_7, respectively, with the conditions for mass equivalence $m_8^a+m_8^b=m_8$ and $m_8^ap_8=m_8^b(a_3-p_8)$.

Since the linkage is based on a 3-DoF principal vector linkage, it has three relative DoFs which can be analyzed individually [4]. Figure 3a illustrates the first relative DoF θ_1 where links A_2A_3 and A_3A_0 are immobile while link A_0A_1 solely rotates about A_0. Then link $P_{43}B_7$ solely rotates about P_{43} while links $P_{12}B_6$ and B_6S solely translate. The linear momentum L_1 of this motion can be written with respect to reference frame x_1y_1, which is aligned with A_0A_1 as illustrated, and must equal the linear momentum of the total mass moving with S, which writes:

$$\frac{L_1}{\dot{\theta}_1}=\begin{bmatrix} m_1e_1+m_2^al_1+(m_5+m_7+m_8^a)a_1+m_6(a_2-p_6)+m_8^ba_2 \\ 0 \end{bmatrix}=\begin{bmatrix} m_{tot}a_1 \\ 0 \end{bmatrix} \tag{2}$$

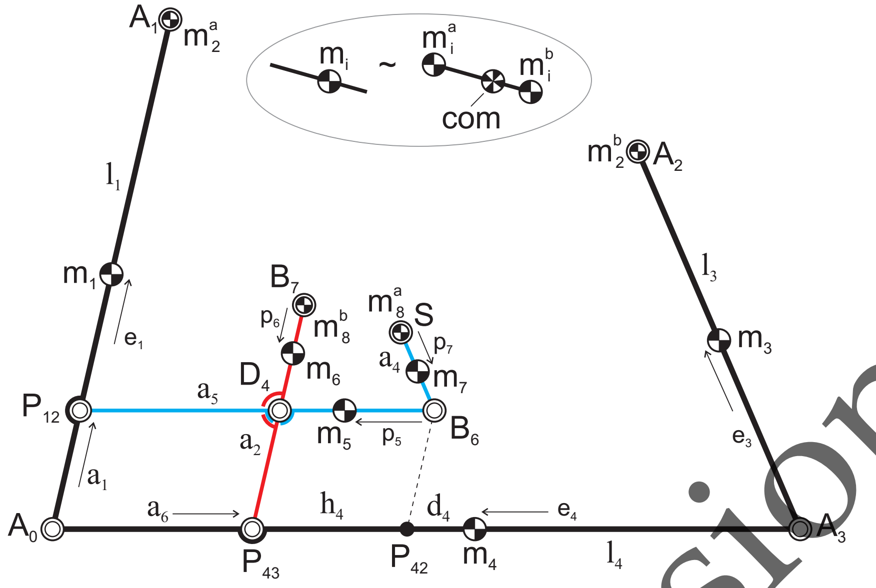

Fig. 2 To derive the force balance conditions, links A_1A_2 and SB_7 are modeled with equivalent masses m_2^a, m_2^b, m_8^a, and m_8^b in joints A_1, A_2, S, and B_7, respectively. The closed chains then are modeled as mass equivalent open chains.

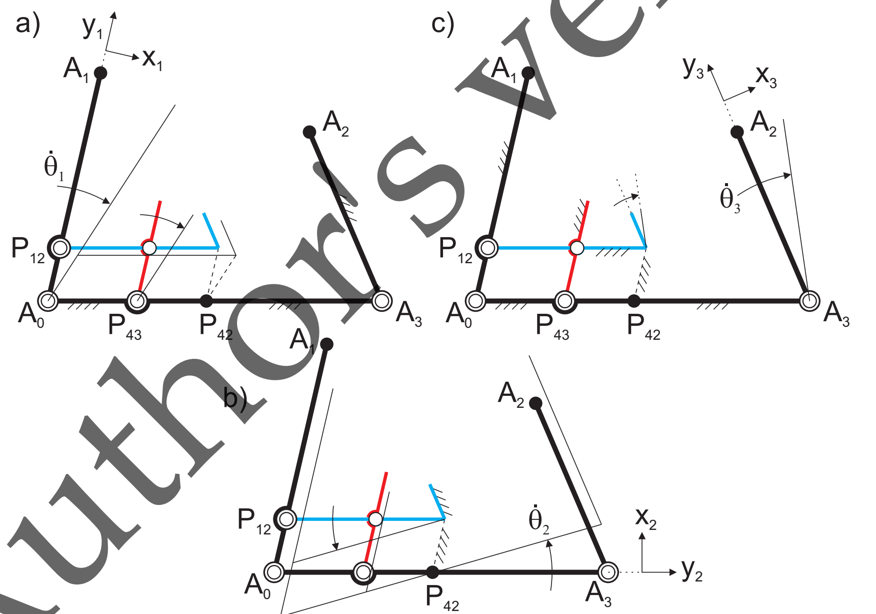

Fig. 3 The force balance conditions are derived from the linear momentum equations of each relative motion individually: (a) DoF 1, (b) DoF 2, and (c) DoF 3

with total mass $m_{tot} = m_1 + m_2 + m_3 + m_4 + m_5 + m_6 + m_7 + m_8$ and the equivalent masses $m_2^a = m_2(1 - e_2/l_2)$, $m_8^a = m_8(1 - p_8/a_3)$, and $m_8^b = m_8 p_8/a_3$.

Figure 3b illustrates the second DoF θ_2 where link A_0A_3 rotates about P_{42}, link $P_{12}B_6$ rotates about B_6, link SB_6 is immobile and the other three links solely translate. The linear momentum L_2 of this motion can be written with respect to reference frame x_2y_2, which is aligned with A_0A_3 as illustrated. Since for this motion the total mass in S is not moving, the linear momentum must equal zero and is written as:

$$\frac{L_2}{\dot{\theta}_2} = \begin{bmatrix} -(m_1 + m_2^a)a_5 + (m_2^b + m_3)(l_4 - a_5) + m_4 d_4 - m_5 p_5 - (m_6 + m_8^b)h_4 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{3}$$

with $m_2^b = m_2 e_2/l_2$ and $d_4 = l_4 - a_5 - e_4$.

Figure 3c illustrates the third DoF θ_3 where links SB_6 and A_2A_3 solely rotate about B_6 and A_3, respectively, and all other links are immobile. The linear momentum L_3 of this motion can be written with respect to reference frame x_3y_3, which is aligned with A_2A_3 as illustrated, and must equal the linear momentum of the total mass moving in S writing:

$$\frac{L_3}{\dot{\theta}_3} = \begin{bmatrix} m_2^b l_3 + m_3 e_3 + m_7(a_4 - p_7) + m_8^a a_4 \\ 0 \end{bmatrix} = \begin{bmatrix} m_{tot} a_4 \\ 0 \end{bmatrix} \tag{4}$$

From the linear momentum equations the three force balance conditions are readily obtained as:

$$\begin{aligned} m_1 e_1 + m_2^a l_1 + (m_5 + m_7 + m_8^a)a_1 + m_6(a_2 - p_6) + m_8^b a_2 - m_{tot} a_1 &= 0 \\ -(m_1 + m_2^a)a_5 + (m_2^b + m_3)(l_4 - a_5) + m_4 d_4 - m_5 p_5 - (m_6 + m_8^b)h_4 &= 0 \\ m_2^b l_3 + m_3 e_3 + m_7(a_4 - p_7) + m_8^a a_4 - m_{tot} a_4 &= 0 \end{aligned} \tag{5}$$

and after substituting m_{tot}, d_4, and h_4 they can be rewritten as:

$$(m_1 + m_2 + m_3 + m_4 + m_6 + m_8^b)a_1 - m_1 e_1 - m_2^a l_1 - m_6(a_2 - p_6) - m_8^b a_2 = 0 \tag{6}$$

$$\begin{aligned} (m_1 + m_2 + m_3 + m_4 + m_6 + m_8^b)a_5 - (m_2^b + m_3)l_4 - m_4(l_4 - e_4) + m_5 p_5 - & \\ (m_6 + m_8^b)a_6 &= 0 \end{aligned} \tag{7}$$

$$(m_1 + m_2 + m_3 + m_4 + m_5 + m_6 + m_8^b)a_4 - m_2^b l_3 - m_3 e_3 + m_7 p_7 = 0 \tag{8}$$

These three balance conditions together with the conditions for similarity Eq. (1) give in total six equations. This means that there are six dependent parameters to be calculated with the others given. One option can be to use the equations to calculate a_1, a_2, a_3, a_4, a_5, and a_6. The sequence of solving the equations then becomes:

$$
\begin{array}{lll}
I. & a_4 = \frac{m_2^b l_3 + m_3 e_3 - m_7 p_7}{m_1+m_2+m_3+m_4+m_5+m_6+m_8^b} & \text{(from Eq. (8))} \\
II. & a_3 = \frac{l_2}{l_3} a_4 & \text{(from Eq. (1))} \\
III. & a_1 = \frac{m_1 e_1 + m_2^a l_1 - m_6 p_6 + (m_6+m_8^b)\frac{l_1}{l_3} a_4}{m_1+m_2+m_3+m_4} & \text{(from Eq. (6))} \\
IV. & a_2 = \frac{l_1}{l_3} a_4 + a_1 & \text{(from Eq. (1))} \\
V. & a_5 = \frac{(m_2^b+m_3) l_4 + m_4(l_4-e_4) - m_5 p_5 - (m_6+m_8^b)\frac{l_4}{l_3} a_4}{m_1+m_2+m_3+m_4} & \text{(from Eq. (7))} \\
VI. & a_6 = a_5 - \frac{l_4}{l_3} a_4 & \text{(from Eq. (1))}
\end{array}
\qquad (9)
$$

3 Spatial inherently balanced linkage with two similar Bennetts

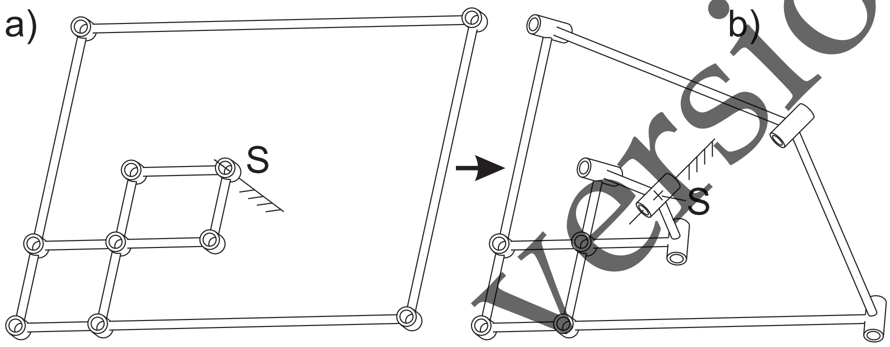

Fig. 4 An inherently balanced double Bennett linkage with common CoM S as fixed joint in (b) can be obtained from the planar version of the linkage where all links are in parallel in (a) by twisting the revolute pairs out of plane. The balance and balance conditions are maintained.

The planar inherently balanced linkage in Fig. 1 can easily be transformed into a spatial inherently balanced linkage. When the two four-bar linkages are made parallelograms with $l_1 = l_3$, $l_2 = l_4$, $a_2 - a_1 = a_4$, and $a_5 - a_6 = a_3$ then the resulting planar linkage becomes as shown in Fig. 4a. With the conditions for similarity Eq. (1) and the balance conditions Eqs. (6-8) the common CoM S is stationary in the fixed pivot for all motion. Then by twisting the revolute pairs of each similar four-bar out of plane according to the Bennett conditions for which the mechanism remains mobile [1], the spatial inherently balanced linkage in Fig. 4b is obtained. Twisting the revolute pairs does not affect the balance and balance conditions, they remain exactly the same with equal values as for the planar version. This is since the principal vectors on which the linkage is based, do not change and are always valid for spatial motion [3]. Only of importance is that the Bennett conditions for the inner and outer four-bar are chosen equal such that the inherently balanced linkage has two similar 4R four-bar Bennett linkages. The parallelogram connecting the two Bennett four-bars remains planar.

Although in this paper all links were assumed mass symmetric, in the out-of-plane direction the links and linkage does not need to be mass symmetric. All links can have a general mass distribution in this direction, comparable to the planar linkage where the out-of-plane mass distribution does not affect the force balance as well. For any out-of-plane mass distribution the common CoM S will be a stationary point on the rotational axis of the fixed joint. In practice this is useful, since especially for producing Bennett linkages with sufficient range of motion without intersecting links, advanced link designs are needed. It is also possible to have all links in this paper have a general CoM, i.e. without mass symmetry in any direction. The balance conditions can be derived with the same shown approach, however it will be more extensive. Then also in general P_{12} and P_{43} will be located off their line through the link joints.

Since twisting the revolute pairs does not affect the force balance, the inherently balanced double Bennett linkage has potential as a reconfigurable and deployable inherently balanced linkage. By actuating the twists the linkage can be altered from planar to spatial, vice versa.

4 Conclusions

In this paper it was shown how a spatial inherently balanced double Bennett linkage could be obtained from a planar inherently balanced linkage of two similar 4R four-bar linkages connected with a parallelogram. Twisting the revolute pairs out of plane according the Bennett conditions does not affect the balance and the balance conditions. The resulting linkage consists of two similar Bennett linkages connected with a planar parallelogram. The balance conditions were derived from the planar linkage by linear momentum equations of each relative degree of freedom individually, showing that this method applies also to inherently balanced linkages that are based on multiple sets of principal vectors.

The approach in this paper can be regarded as an approach to synthesize spatial inherently balanced mechanisms from planar linkage architectures with the advantage to not have to consider complex spatial kinematics. As a bonus, it leads spontaneously to a variety of inherently balanced reconfigurable and deployable spatial linkages, as was shown for the example in this paper.

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