A New Direct Position Analysis Solution for an Over-constrained Gough-Stewart Platform

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Abstract. Recently, the authors presented a new over-constrained manipulator with six degrees of freedom, based on a modified Gough-Stewart platform, and a solution for its direct position analysis. In this paper, a different solution is proposed based on a different parameterization that leads to a reduced system of four closure equations. The new method simplifies the analytical derivation and the geometrical interpretation of the results.

Key words: Modified Gough-Stewart platform, direct position analysis, reduced equation system

1 Introduction

Several types of Gough-Stewart (GS) platforms were proposed in the literature [1]. A new mechanism has been recently presented [2], together with its direct position analysis (DPA). The new manipulator is an evolution of a previous type of GS platform [3] and has several interesting characteristics [2] with respect to the classic GS: it features a lower number of kinematic pairs thus simplifying the mechanical design; it is an over-constrained mechanism, giving the opportunity to remove clearance in kinematic pairs; it features a larger workspace, free from kinematic singularities for practical mechanism dimensions.

In this paper a new solution of the DPA is presented. The closure equations of the mechanism have been found relying upon a technique, known as “open loop chain”, that was presented in [4] and used to solve the DPA of many mechanisms. Differently from the classical approach, we show in this paper that the core of the DPA can be reduced to a system of four equations in four unknowns. The full analytical derivation is reported here and the new solution is discussed.

2 Modified Gough-Stewart manipulator

A full description of the new mechanism and of its characteristics is presented in [2]: only its general features are reported here for the sake of clarity. The mechanism (Fig. 1) is composed of a mobile platform (1) (defined by the points C_i, i=1,2,3, that define the plane σ), with six degrees-of-freedom with respect to the fixed base (2) (defined by the points $A_{i,j}$, i=1,2,3, j=1,2), that are connected by means of three kinematic chains i=1,2,3 (Fig. 2), defined by the points $A_{i,1}$, $A_{i,2}$, $B_{i,2}$, $B_{i,1}$. The mobile platform is connected to the upper link $B_{i,2}B_{i,1}$ of each kinematic chain by the universal joints centered at points C_i . The two axes of the universal joint must not be parallel to the normal to the plane γ_i passing through the points C_i, $A_{i,1}$, $A_{i,2}$, as to avoid redundancy. In each kinematic chain, $B_{i,j}$ and $A_{i,j}$ denote the connection points of the linear actuators with the upper link and the fixed base, by revolute and universal joints respectively. These joints must comply with some geometrical conditions: the revolute joint axes and the mobile axes of two universal joints must be parallel, while the other universal joint axes must be collinear (Fig. 2). Because of the linear constraints, the i-th kinematic chain lies on the plane γ_i for any configuration of the mobile platform.

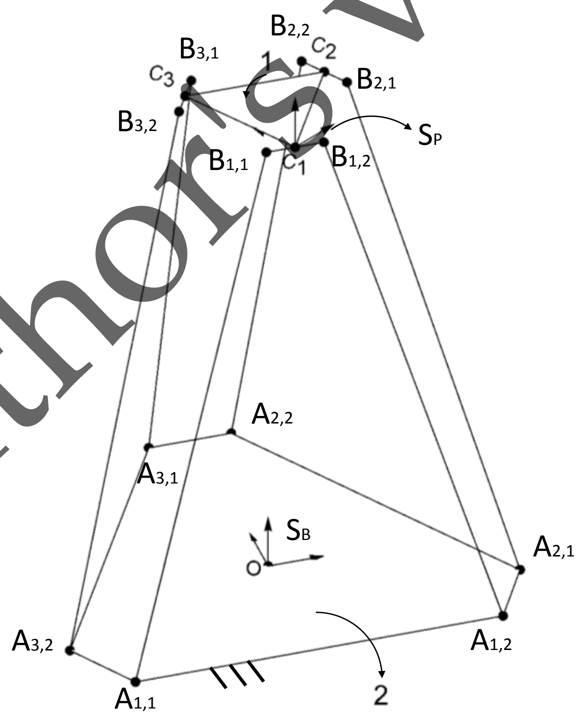

Fig. 1 Schematic representation of the manipulator.

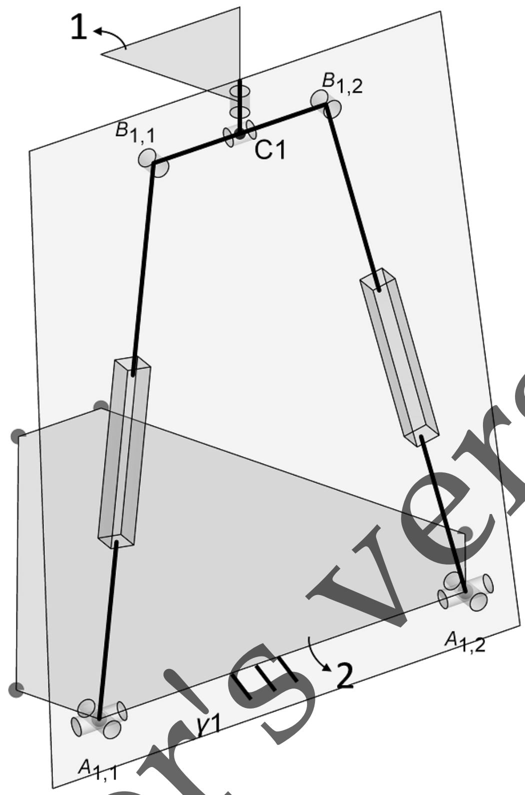

Fig. 2 Detailed representation of the first kinematic chain.

3 Direct position analysis

The DPA problem is to find the configuration of the mobile platform, given the lengths of the legs. For the sake of simplicity, the points $A_{i,j}$ of the base will be considered on the same plane, though the mechanism allows a more general geometry also. However, the DPA presented in this paper can be easily generalized to the general geometry by a few adjustments. Two Cartesian coordinate systems are defined (Fig. 1). The first one (S_B) is attached to the fixed base: its center is located at the centroid O of the fixed base, x axis parallel to the vector $\mathbf{A}_{1,1}\mathbf{A}_{1,2}$, z axis is orthogonal to the plane on which the fixed base lies, y axis as a consequence. The second coordinate system (S_P) is attached to the mobile platform: it has center on the point C_1, y axis coincident with the direction $\mathbf{C}_1\mathbf{C}_3$, z axis orthogonal to the plane σ, x axis as a consequence. The mechanism geometry is defined as follows: $\mathbf{a}_{i,j}$ is the position vector of the point $A_{i,j}$ in S_B; $\mathbf{c}_i$ is the position vectors of the point C_i in S_P; $\mathbf{r}_i$ is the vector that identifies the frame link of the i-th kinematic chain (i.e., $\mathbf{r}_i=\mathbf{A}_{i,2}\mathbf{A}_{i,1}$); $l_{i,j}$ is the length of the j-th link of the i-th kinematic chain (i.e., $l_{i,j}=\|\mathbf{A}_{i,j}\mathbf{B}_{i,j}\|$); $\mathbf{k}$ is the unit vector normal to the plane σ (i.e., the unit vector of the z axis of the S_P); $\mathbf{u}_i$ is the unit vector normal to the plane γ_i; $\mathbf{t}_i$ is the unit vector that defines the direction of the vector $\mathbf{B}_{i,1}\mathbf{B}_{i,2}$ (i.e., $\mathbf{t}_i=\mathbf{B}_{i,1}\mathbf{B}_{i,2}/\| \mathbf{B}_{i,1}\mathbf{B}_{i,2}\|$), and l is its norm (i.e., $l=\| \mathbf{B}_{i,1}\mathbf{B}_{i,2}\|$).

The position of the points C_1 and $B_{1,j}$ can be described with respect to the S_B by the four parameters ψ_i (i=1,2,3,4) (Fig. 3): ψ_1 is the angle between the vector $\mathbf{r}_1$ and the vector $\mathbf{A}_{1,1}\mathbf{B}_{1,1}$, ψ_2 is the angle between the plane γ_1 and the plane identified by the fixed base, ψ_3 is the angle between the vectors $\mathbf{k}$ and $\mathbf{u}_1$, and ψ_4 is the angle between the unit vector obtained as the cross product between the vector $\mathbf{k}$ and $\mathbf{t}_1$ and the y axis of the S_P. In particular, the position of the point C_1 can be expressed as a function of ψ_1 and ψ_2 and the lengths $l_{1,1}$ and $l_{1,2}$ only, as it will be clarified further on. Moreover, it is worth noting that the kinematic chain $A_{1,1}B_{1,1}B_{1,2}A_{1,2}$ is a four bar mechanism if the prismatic joints are locked, and the angle θ (i.e., the angle between the upper and the lower links) can be expressed as a function of the angle ψ_1 and the lengths $l_{1,1}$ and $l_{1,2}$, according to the well-known relation [5]:

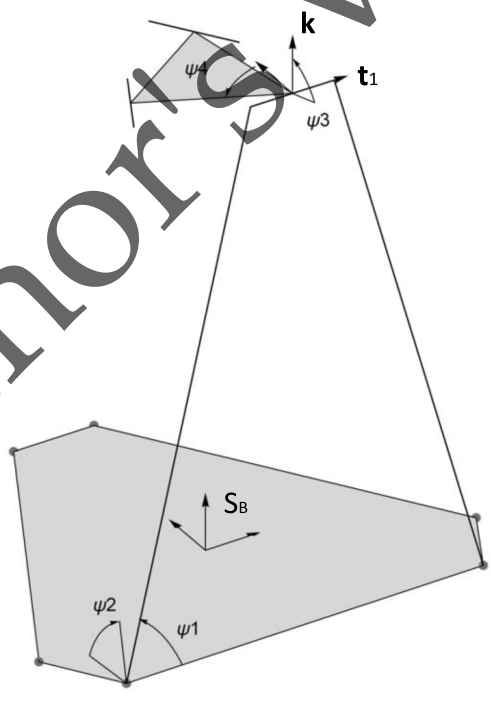

Fig. 3 Representation of the four parameters used for the DPA.

$$\theta = 2\tan^{-1}\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right) \tag{1}$$

where

$$\begin{aligned} a &= -h_1 + (1+h_2)\cos(\psi_1) + h_4 \\ b &= -2\sin(\psi_1) \\ c &= h_1 - (1-h_2)\cos(\psi_1) + h_4 \end{aligned} \tag{2}$$

and

$$h_1 = \frac{r_1}{l_1}, \quad h_2 = \frac{r_1}{l}, \quad h_4 = \frac{-r_1^{\,2} - l_1^{\,2} - l^2 + l_2^{\,2}}{2\cdot l\cdot l_1} \tag{3}$$

The position vector of the point C_1 can be thus written as:

$$\mathbf{OC}_1 = \mathbf{a}_{1,1} + \mathbf{A}_{1,1}\mathbf{B}_{1,1} + \frac{\mathbf{B}_{1,1}\mathbf{B}_{1,2}}{2} \tag{4}$$

Where the vector $\mathbf{B}_{i,1}\mathbf{B}_{i,2}$, can be express in S_B as:

$${}^{B}(B_{1,1}B_{1,2}) = l\begin{pmatrix}\cos\theta \\ \sin\theta\cos\psi_2 \\ \sin\theta\sin\psi_2\end{pmatrix}, \quad {}^{B}(A_{1,1}B_{1,1}) = l_{1,1}\begin{pmatrix}\cos\psi_1 \\ \sin\psi_1\cos\psi_2 \\ \sin\psi_1\sin\psi_2\end{pmatrix} \tag{5}$$

The position vector $\mathbf{OC}_i$ (i=2,3) can be expressed as a function of the four parameters as follows:

$${}^{B}(OC_i) = {}^{B}(OC_1) + {}^{B}\mathbf{R}_P\,{}^{P}c_i \quad i = 2,3 \tag{6}$$

where

$${}^{B}\mathbf{R}_P = \mathbf{R}_{\mathbf{r}_1}(\psi_2)\mathbf{R}_{\mathbf{u}_1}(\theta)\mathbf{R}_{\mathbf{t}_1}(\psi_3)\mathbf{R}_{\mathbf{k}}(\psi_4) \tag{7}$$

In (7), each $\boldsymbol{R}$ is the 3x3 orthonormal matrix that represents a rotation defined by the angle in brackets about the axis specified in the subscript.

The expression of the point $B_{i,j}$ (i=2,3; j=1,2) with respect to the S_B can be determined without adding new variables. In particular, it is worth noting that the direction $\mathbf{t}_i$ is obtained as the intersection between the two planes σ and γ_i (Fig. 4). In fact, the joint centered in C_i allows the upper link of the i-th kinematic chain to

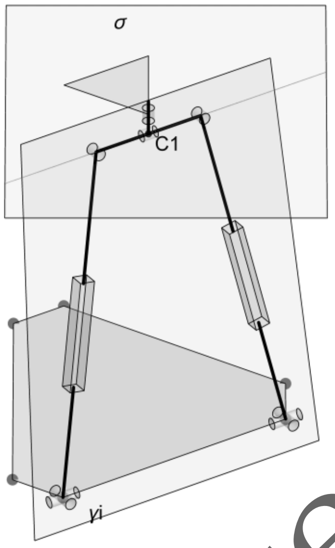

Fig. 4 Definition of the line that passes through the upper link of the kinematic chain.

rotate about the axis **k**, so as the direction $\mathbf{t}_i$ lies on the plane σ. Furthermore, the direction $\mathbf{t}_i$ lies on the plane γ_i, since the vector $\mathbf{B}_{i,1}\mathbf{B}_{i,2}$ identifies the upper link of the i-th kinematic chain. Thus, the vector $\mathbf{t}_i$ can be found as the cross product between **k** and $\mathbf{u}_i$:

$$\mathbf{t}_i = \frac{\mathbf{k} \times \mathbf{u}_i}{|\mathbf{k} \times \mathbf{u}_i|} \tag{8}$$

Since:

$$\mathbf{u}_i = \frac{\mathbf{r}_i \times \mathbf{A}_{i,1}\mathbf{C}_i}{|\mathbf{r}_i \times \mathbf{A}_{i,1}\mathbf{C}_i|} \tag{9}$$

Finally, $\mathbf{t}_i$ can be expressed without adding new variables:

$$\mathbf{t}_i = \frac{\mathbf{k} \times (\mathbf{r}_i \times \mathbf{A}_{i,1}\mathbf{C}_i)}{|\mathbf{k} \times (\mathbf{r}_i \times \mathbf{A}_{i,1}\mathbf{C}_i)|} \tag{10}$$

where

$$\mathbf{A}_{i,1}\mathbf{C}_i = \mathbf{OC}_i - \mathbf{a}_{i,1} \tag{11}$$

The position vector $B_{i,j}$ with respect to the S_B can be written as:

$$\mathbf{OB}_{i,j} = \mathbf{a}_{i,j} + \mathbf{A}_{i,j}\mathbf{B}_{i,j} = \mathbf{OC}_i \pm l\frac{\mathbf{t}_i}{2} \tag{12}$$

Thus, a system of four equations in the four unknowns ψ_n, n=1,…,4, can be obtained:

$$(\mathbf{A}_{i,j}\mathbf{B}_{i,j})^T(\mathbf{A}_{i,j}\mathbf{B}_{i,j}) = l_{i,j}{}^2 = (\mathbf{OC}_i \pm l\frac{\mathbf{t}_i}{2} - \mathbf{a}_{i,j})^T(\mathbf{OC}_i \pm l\frac{\mathbf{t}_i}{2} - \mathbf{a}_{i,j}) \tag{13}$$

$$i = 2,3; \quad j = 1,2;$$

This system represents the final solution of the DPA, since it makes it possible to obtain the values of the parameters ψ_n that describe the platform pose, when the mechanism geometry and the actuator lengths are given.

4 Numerical example

As an example, a specific geometry of the mechanism is considered in this section and its configuration is determined with the proposed DPA method for three representative combinations of actuator lengths. The points $A_{i,j}$ are on a circle with diameter d_b=840mm; position vectors of consecutive points $A_{i,j}$ belonging to different kinematic chains form an angle of $\varphi=\pi/9$ (Fig. 5); the points C_i of the mobile platform form an equilateral triangle inscribed in a circumference of diameter d_p=280mm; the length of the upper link is l=100mm. In the first considered combination the actuators have all the same length (corresponding to the initial configuration of the platform); in the second one, the actuators have the same length three by three that provide a configuration in which the platform is rotated about the z axis); in the third configuration, the actuators have length that provide a configuration in which the platform is rotated about the y axis.

Table 1. Actuator lengths and corresponding values of the configuration parameters at the three considered mechanism poses.

$[l_{1,1}\ l_{1,2}\ l_{2,1}\ l_{2,2}\ l_{3,1}\ l_{3,2}]$	Ψ_1	Ψ_2	Ψ_3	Ψ_4
[958.6, 958.6, 958.6, 958.6, 958.6,958.6]	1.33736	1.43097	-1.43097	0.523599
[891.6, 847.8, 891.6, 847.8, 891.6,847.8]	1.21866	1.37925	-1.37925	1.183130
[973.3 969.8 1004.2 1002.9 939.5 941.6]	1.17517	1.43822	-1.47862	0.438011

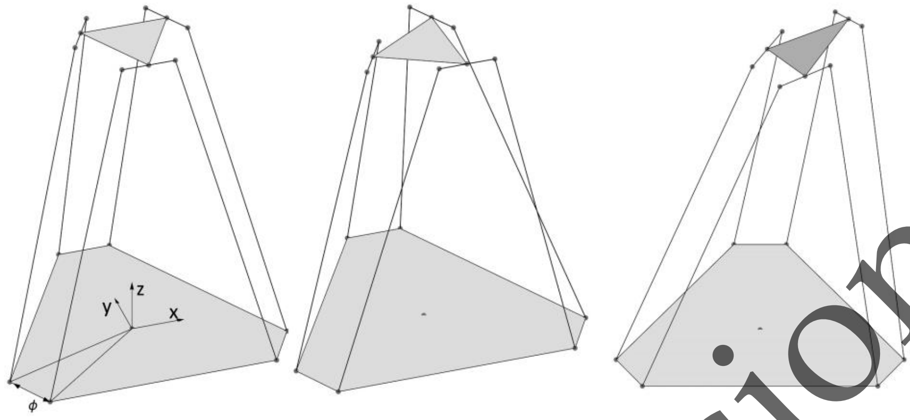

Fig. 5 The three considered mechanism configurations.

5 Conclusions

In this paper, a new solution for the DPA of a recently proposed over-constrained parallel manipulator is presented. The mechanism is a modified version of the Gough-Stewart manipulator, in which the platform is connected to the base by three kinematic chains that behave as four bar linkages when the actuator lengths are fixed. The new DPA solution is based on a parameterization that leads to a system of four equations in four unknowns, thus reducing the classic system of six equations in six unknowns. This parameterization makes it possible to represent the platform pose through the configuration of a single kinematic chain, thus simplifying the analytical derivation and the geometrical interpretation of the results.

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