Control Based on Linear Algebra for Mobile Manipulators

Víctor H. Andaluz, Edison R. Sásig, William D. Chicaiza and Paola M. Velasco

Universidad de las Fuerzas Armadas ESPE Sangolquí, Ecuador, e-mail: {vhandaluz1,erssigs, wdchicaiza1,pmvelasco1}@espe.edu.ec

Abstract. This paper presents a control algorithm based on linear algebra for trajectory tracking of mobile manipulator robots. The proposed control algorithm considers the kinematics of the robot, which is approximated by the Euler method, the control actions for an optimal operation of the system are obtained solving a system of linear equations. In addition, the stability of the system is analyzed by concepts of linear algebra, where it is shown that the control error tends asymptotically to zero. Simulation results show the good performance of the proposed control system.

Key words: Mobile Manipulator, Model, Controller design, Linear Algebra, Numerical Methods.

1 Introduction

Mobile manipulator is nowadays a widespread term that refers to robots built with a robotic arm mounted on a mobile platform. This kind of system, which is usually characterized by a high degree of redundancy, combines the manipulability of a fixed-base manipulator with the mobility of a wheeled platform. Such systems allow the most usual missions of robotic systems which requiere both locomotion and manipulation abilities. They are useful in multiple applications in different industrial and productive fields, such as mining, construction, rescue missions or for people assistance [1,2].

In the literature different control strategies have been proposed. Work [3] solves the trajectory tracking problem by combining neural networks and robust control. The nonlinear mapping characteristic of neural networks and robust control are integrated in an adaptive control algorithm for mobile manipulator robots with nonlinearities, perturbations and non-holonomic constraints all at the simulation level. The project carried out in [4] suggests a fuzzy PD controller to adjust the parameters in line depending on the state of the dynamic system. Other advanced control strategies are implemented for example in [5] introduces a constrained predictive control algorithm for a holonomic mobile manipulator robot. Restrictions such as acceleration, velocity, position, and avoiding obstacles are considered.

The control based on linear algebra is a novel technique whose main feature is that there is no need for complex calculations to achieve control signal and simplicity in performing mathematical operations [6, 7, 8]. In addition, the algorithm is easy to understand and implement, allowing direct adaptation to any microcontroller without the use of an external computer [9]. Because it is not a complex algorithm it can run on controllers with low processing capacity [10].

In this work the control based on linear algebra is applied for tasks of tracking of trajectories in mobile manipulator robots. The controller is based on the kinematics of the system formed by a robotic arm mounted on a mobile platform. The structure of the control law consists of a particular solution that meets the raised objective. Additionally, the stability is proved through linear algebra concepts. To validate the proposed control algorithms, experimental results are included and discussed.

This article is organized into 5 Sections. Section 2 presents the robot kinematic model for the mobile manipulator robot. The design of the control algorithm is presented in Section 3. The discussion of results is shown in Section 4, and finally the conclusions of the paper are presents in Section 5.

2 Mobile Manipulator Modeling

In this section, the kinematic model of the mobile manipulator is presented. For this purpose, the mobile manipulator configuration is defined by a vector $\mathbf{q} = [q_1 \quad q_2 \quad \cdots \quad q_n]^T = [\mathbf{q}_p^T \quad \mathbf{q}_a^T]^T$ of n independent coordinates called generalized coordinates of the mobile manipulator, where $\mathbf{q}_a$ represents the generalized coordinates of the arm, and $\mathbf{q}_p$ the generalized coordinates of the mobile platform. The location of the end-effector of the mobile manipulator is given by the *m*-dimensional vector of operational coordinates $\mathbf{h} = [h_1 \quad h_2 \quad \cdots \quad h_m]^T$ [11].

The kinematic model of a mobile manipulator gives the location of the end-effector as a function of the robotic arm configuration and the platform location [11]. The instantaneous kinematic model of a mobile manipulator gives the derivative of its end-effector location as a function of the derivatives of both the robotic arm configuration and the location of the mobile platform.

$$\dot{\mathbf{h}}(t) = \mathbf{J}(\mathbf{q})\mathbf{v}(t) \tag{1}$$

where $\dot{\mathbf{h}} = [\dot{h}_1 \quad \dot{h}_2 \quad \cdots \quad \dot{h}_m]^T$ is the vector of end-effector velocity, $\mathbf{v} = [v_1 \quad v_2 \quad \cdots \quad v_{\delta_n}]^T = [v_p^T \quad v_a^T]^T$ is the vector of mobile manipulator velocities in which contains the linear and angular velocities of the mobile platform and contains the joint velocities of robotic arm and $\mathbf{J}(\mathbf{q})$ is the Jacobian matrix that defines a linear mapping between the vector of the mobile manipulator velocities $\mathbf{v}(t)$ and the vector of the end-effector velocity [12].

3 Controller Design

In this section, the control law based on linear algebra theory and numerical methods is presented. Futhermore, the stability is proved through linear algebra concepts.

3.1 Kinematic Controller

Through the Euler's approximation of the kinematic model of the mobile manipulator (1), the following kinematic model discrete is obtained

$$\mathbf{h}(k+1) = \mathbf{h}(k) + T_0 \mathbf{J}(\mathbf{q}(k))\mathbf{v}(k) \tag{2}$$

where, values of $\mathbf{h}$ at the discrete time $t = kT_0$ will be denoted as $\mathbf{h}(k)$, T_0 is the sample time, and $k \in \{0,1,2,3,4,5...\}$. Next by the Markov property and to adjusting the performance of the proposed control law [13], the states vector $\mathbf{h}(k+1)$ is replaced by,

$$\mathbf{h}(k+1) = \mathbf{h_d}(k+1) - \mathbf{W}(\mathbf{h_d}(k) - \mathbf{h}(k)) \tag{3}$$

where, $\mathbf{W}$ is a diagonal matrix and its values are obtained in Sect. **3.2**, these constants satisfy $0 < \mathbf{diag}(w_{hx}, w_{hy}, w_{hz}) < 1$, allowing to reduce the variations in state variables and $\mathbf{h_d}$ is the desired trajectory.

Then, from (2) and (3), the following system of linear equations is obtained, which allows at each sampling instant to calculate the control actions.

$$\mathbf{Jv} = \mathbf{b} \tag{4}$$

where $\mathbf{v} = \begin{bmatrix} u(k) & \omega(k) & \dot{q}_1(k) & \dot{q}_2(k) & \dot{q}_3(k) \end{bmatrix}^T$ and

$$\mathbf{b} = \frac{1}{T_0}\begin{bmatrix} h_{xd}\left(k+1\right) - w_{hx}\left(e_{hx}\left(k\right)\right) - h_x\left(k\right) \\ h_{yd}\left(k+1\right) - w_{hy}\left(e_{hy}\left(k\right)\right) - h_y\left(k\right) \\ h_{zd}\left(k+1\right) - w_{hz}\left(e_{hz}\left(k\right)\right) - h_z\left(k\right) \end{bmatrix}$$

From (4), which is a set of three equations with five unknown variables, its solution by least squares is obtained by solving the normal equations.

$$\mathbf{v}_{ref} = \mathbf{J}^T\left(\mathbf{J}\mathbf{J}^T\right)^{-1}\frac{1}{T_0}\begin{bmatrix} h_{xd}\left(k+1\right) - w_{hx}\left(e_{hx}\left(k\right)\right) - h_x\left(k\right) \\ h_{yd}\left(k+1\right) - w_{hy}\left(e_{hy}\left(k\right)\right) - h_y\left(k\right) \\ h_{zd}\left(k+1\right) - w_{hz}\left(e_{hz}\left(k\right)\right) - h_z\left(k\right) \end{bmatrix}. \qquad (5)$$

4 Simulation Results

In order to assess and discuss the performance of the controller based on linear algebra. It's developed a simulation platform for mobile manipulator on Matlab© platform. This is an online simulator, which allows users to view three-dimensional environment navigation of a mobile manipulator.

To check the performance of the control system presents in (5). Two tests are implemented: the first desired trajectory for the end-effector of the mobile manipulator is described by $\mathbf{h}_{\mathrm{d}} = \begin{bmatrix} h_{xd} & h_{yd} & h_{zd} \end{bmatrix}^{\mathrm{T}}$, where $h_{xd} = 0.1t$, $h_{yd} = 0.3\sin\left(0.3t\right)$ and $h_{zd} = 0.5$, the mobile platform starts at $\mathbf{q}_{\mathrm{p}} = \begin{bmatrix} 0\text{ m} & -0.2\text{ m} & 0\text{ rad} \end{bmatrix}^{\mathrm{T}}$; the robotic arm at $\mathbf{q}_{\mathrm{a}} = \begin{bmatrix} 0\text{ rad} & \frac{\pi}{4}\text{ rad} & -\frac{\pi}{2}\text{ rad} \end{bmatrix}^{\mathrm{T}}$, Figures 1 to 4, represent the experimental results.

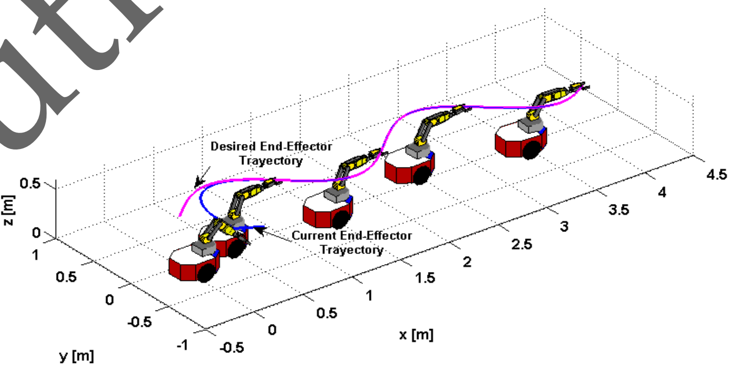

Fig. 1 Stroboscopic movement of the mobile manipulator in the trajectory tracking experiment

Figure 1, shows the desired trajectory and the current trajectory of the end-effector. It can be seen that the proposed controller presents a good performance. Figure 2, shows the evolution of the tracking errors, which remain close to zero, while Figures 3 and 4 show the control actions.

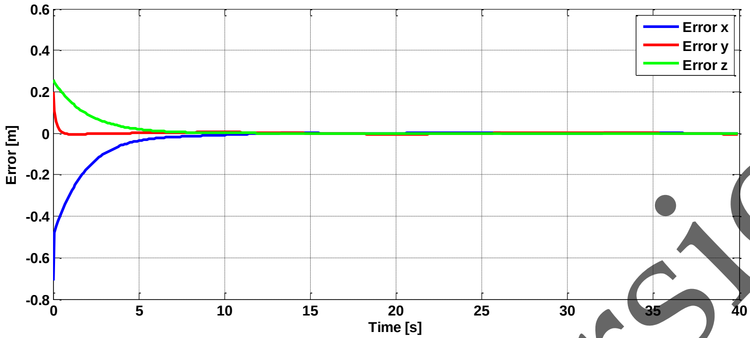

Fig. 2 Control errors of the mobile manipulator

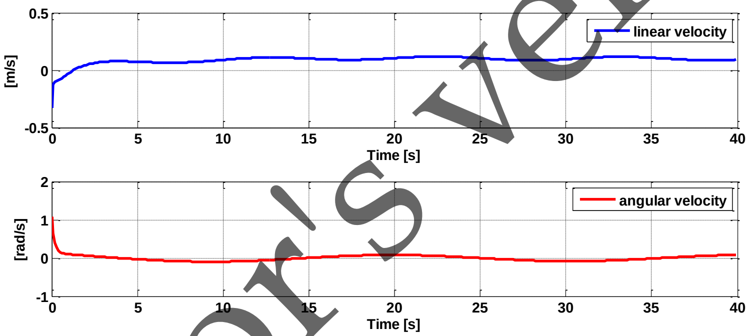

Fig. 3 Velocity commands to the mobile platform

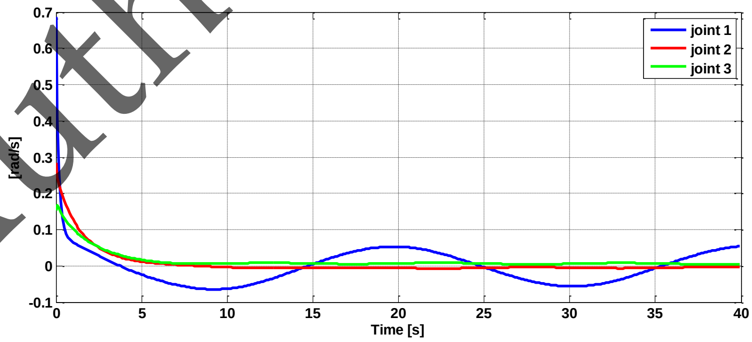

Fig. 4 Joint velocity commands to the robotic arm

The second desired trajectory is described by $\mathbf{h_d} = \begin{bmatrix} h_{xd} & h_{yd} & h_{zd} \end{bmatrix}^{\mathrm{T}}$, where $h_{xd} = 0.7 + \sin(0.4t)$, $h_{yd} = \sin(0.2t)$ and $h_{zd} = 0.37 + 0.1\sin(0.2t)$. In this experiment, the mobile platform starts at $\mathbf{q}_{\mathrm{p}} = \begin{bmatrix} 0.8\ \mathrm{m} & -0.1\ \mathrm{m} & \pi\ \mathrm{rad} \end{bmatrix}^{\mathrm{T}}$; the robotic arm at $\mathbf{q}_{\mathrm{a}} = \begin{bmatrix} 0\ \mathrm{rad} & \frac{\pi}{2}\mathrm{rad} & -\frac{\pi}{2}\ \mathrm{rad} \end{bmatrix}^{\mathrm{T}}$, The following figures illustrate the simulation results.

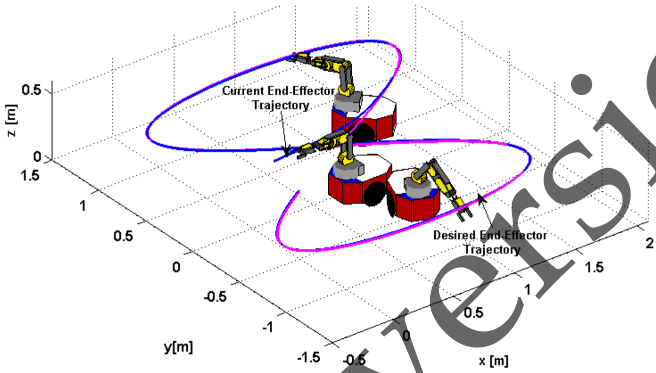

Fig. 5 Stroboscopic movement of the mobile manipulator in the trajectory tracking experiment

Figure 5, shows the desired trajectory and the current trajectory of the end-effector. It can be seen that the proposed controller presents a good performance. Figure 6, shows the evolution of the tracking errors, which remain close to zero, while Figures 7 and 8 show the control actions.

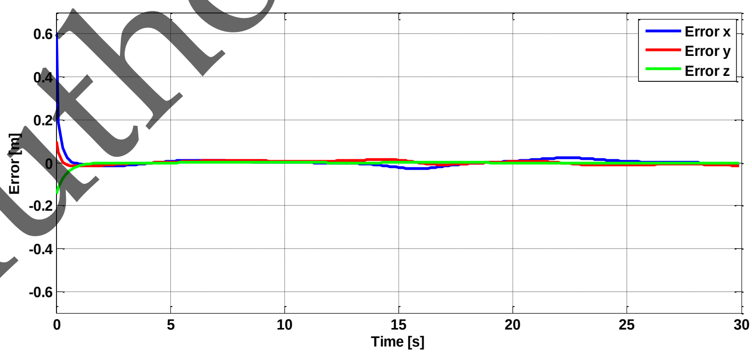

Fig. 6 Control errors of the mobile manipulator

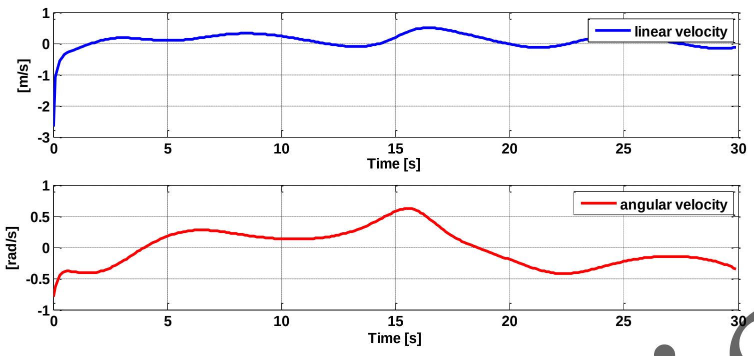

Fig. 7 Velocity commands to the mobile platform

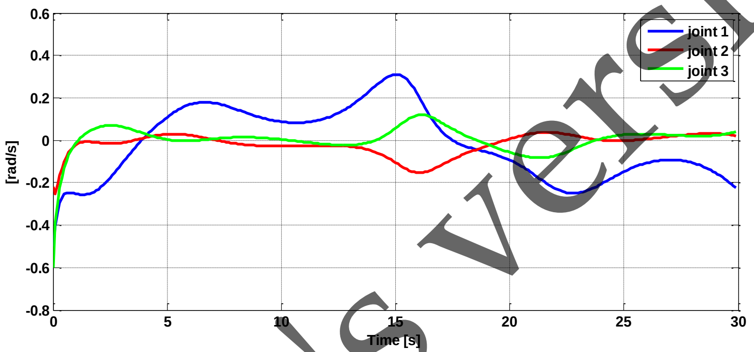

Fig. 8 Joint velocity commands to the robotic arm

6 Conclusions

In this work it was proposed a control algorithm based on concepts of linear algebra and numerical methods for trajectory tasks of mobile manipulators robots. The structure of the control algorithm consists of a particular solution that meets the stated objective. The stability and performance of the proposed control algorithm was demonstrated analytically through concepts of linear algebra. The simulation results obtained show the good performance of the proposed control algorithm.

Acknowledgments The authors would like to thanks to the Consorcio Ecuatoriano para el Desarrollo de Internet Avanzado -CEDIA-, and the Universidad de las Fuerzas Armadas ESPE for financing the project “Tele-Operación Bilateral Cooperativo de Múltiples Manipuladores Móviles – CEPRAIX-2015-05”.

References

1. Padois, V. and Fourquet, J.-Y. and Chiron, P.: Kinematic and dynamic model-based control of wheeled mobile manipulators: a unified framework for reactive approaches. Cambridge University Press, vol. 5, pp. 157 – 173 (2007)
2. Rosales, A. and Scaglia, G. and Mut, V. and Sciascio, F. d.: Navegación de Robots Móviles en entornos no Estructurados utilizando Álgebra Lineal. Revista Iberoamericana de Automática e Informática Industrial RIAI, vol. 6, pp. 79 – 88 (2009)
3. Naijian, C. and Hao, Y. and Xiangdong, H. and Changsheng, A. and Chenglong, T. and Xiangkui, L.: Adaptive robust control of the mobile manipulator based on neural network. IEEE Conference Publications: 2016 IEEE 11th Conference on Industrial Electronics and Applications (ICIEA), pp. 2425 – 2430 (2016)
4. Karray, A. and Feki, M.: Tracking control of a mobile manipulator with fuzzy PD controller. IEEE Conference Publications: 2015 World Congress on Information Technology and Computer Applications (WCITCA), pp. 1 – 5 (2015)
5. Avanzini, G. B. and Zanchettin, A. M. and Rocco, P.: Constraint-based Model Predictive Control for holonomic mobile manipulators. IEEE Conference Publications: 2015 IEEE / RSJ Conferencia Internacional de Robótica Inteligente y Sistemas (IROS), pp. 1473 – 1479 (2015)
6. Scaglia, G. and Rosales, A. and Quintero, L. and Mut, V. and Agarwal, R.: A linear-interpolation-based controller design for trajectory tracking of mobile robots. ELSEVIER: Control Engineering Practice, vol. 18, nº 3, pp. 318 – 329 (2010)
7. Rómoli,S. and Serrano, M. E. and Ortiz, O. A. and Vega, J. R.: Tracking control of concentration profiles in a fed-batch bioreactor using alinear algebra methodology. Elsevier Ltd, vol. 57, pp. 162 – 171 (2015)
8. Gandolfo, D. and Rosales, C. and Patiño, D. and Scaglia, G. and Jordan, M.: Trajectory tracking control of a pvtol aircraft based on linear algebra theory. Asian Journal of Control, pp. 1849 – 1858 (2014)
9. Rosales, A. and Scaglia, G. and Mut, V. and Sciascio, F. d.: Trajectory tracking of mobile robots in dynamic environments—a linear algebra approach. Cambridge University, vol. 27, nº 7, pp. 981–997 (2009)
10. Špinka, O. and Hanz´lek, Z.: Energy-Aware Navigation and Guidance Algorithms for Unmanned Aerial Vehicles. IEEE Conference Publications: 2011 IEEE 17th International Conference on Embedded and Real-Time Computing Systems and Applications, vol. 2, pp. 83 – 88 (2011)
11. Bayle, B. and Fourquet, J.-Y. and Renaud, M.: Kinematic Modelling of Wheeled Mobile Manipulators. IEEE: International Conference on Robotics &Automation, vol. 1, pp. 69 – 74 (2003)
12. Andaluz, V. and Roberti, F. and Toibero, J. M. and Carelli, R.:Adaptive unified motion control of mobile manipulators. ELSERVIER: Control Engineering Practice, vol. 20, pp. 1337-1352, 2012.
13. Romoli, S. and Scaglia, G. and Serrano, M. E. and Godoy, S. and Ortiz, O. A. and Vega, J. R.: Control of a Fed-Batch Fermenter Based on a Linear Algebra Strategy. IEEE Journals & Magazines, pp. 1206 – 1213 (2014)