Robust multi-objective design optimization of the 3-UPU TPM based on the GA-Krawczyk method

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Abstract. This paper deals with the robust design optimization of the 3-UPU translational parallel manipulator. An approach, that regroups the genetic algorithm multi-objective optimization and the Krawczyk operator (GAMOK), is used to represent the optimal design vector of parameters and their uncertainties. This optimization leads to minimize the position error and relax the parameters intervals of tolerance. Based on this GAMOK algorithm, the designer can pick out the optimal design vector according to the desired accuracy in the workspace of the manipulator.

Key words: Interval analysis, Krawczyk operator, Genetic algorithm, uncertainties, optimization.

1 Introduction

Parallel manipulators have many advantages such as, greater rigidity, higher stiffness and essentially higher accuracy compared to serial ones. There are several types of parallel manipulators; the translational robot, the rotational robot and the mixed ones [9, 10]. The position error of parallel manipulator caused by design parameters uncertainties cannot be neglected. Therefore, it is quite important to optimize the design parameters and their uncertainties as function of the robot performances. Several optimization methods have been used. Genetic algorithm (GA) is an evolutionary algorithm inspired from natural evolution, used to solve optimization problems [1]. The main advantages of the genetic algorithm are the capability to escape local optima and its powerful searching ability. Laribi et *al*. proposed a combined GA-fuzzy algorithm in order to find the optimal dimensions of a five bare mechanism for a desired closed curve [5]. El Kribi et *al*. developed a multi-objective genetic algorithm of a mechatronic system with continuous and discrete variables [2].

In this paper, an approach that couples the genetic algorithm and the "Krawczyk" method used for the robust design optimization of the 3-UPU TPM. In fact, the robot optimal design vector is given simultaneously by the design parameters nominal values (DPNV) and the parameters uncertainties .The rest of this paper is organized as follow: in Section 2, the structure of the manipulator is described and its kinematic modeling is presented. In section3, an hybrid GA-"Krawczyk" algorithm is presented. A case study is finally presented which shows the efficiency of the proposed algorithm. In Section4, some concluding remarks are presented.

2 Architecture of the manipulator

The 3-UPU translational parallel manipulator (fig.1) is composed of three kinematic chains of type UPU (U and P stand for universal and prismatic joint respectively) that connect the base to the platform. This manipulator has extensible legs which are connected to the base by universal joints. Each universal joint comprises two revolute pairs with intersecting and perpendicular axes. To restrict the platform motions to only translations, the following conditions have to be satisfied [8,4]: the axes of the two intermediate revolute pairs are parallel to each other and the axes of the two ending revolute pairs are parallel to each other.

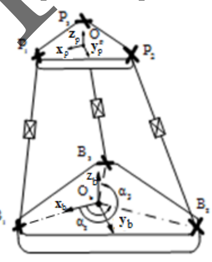


Fig. 1 Architecture of the 3-UPU translational parallel manipulator

2.1. Modeling of the 3-UPU translational manipulator

Let $\mathbf{S}_b(O_b, \mathbf{x}_b, \mathbf{y}_b, \mathbf{z}_b)$ and $\mathbf{S}_p(O_p, \mathbf{x}_p, \mathbf{y}_p, \mathbf{z}_p)$ two reference systems fixed on the base and the platform, respectively. The origin O_b (O_p) corresponds to the reference point of the base (platform). The first axis $\mathbf{x}_b$ goes through point B_1; $\mathbf{z}_b$ axis is normal to the plane defined by the points B_i and pointing from the base to the platform, while $\mathbf{y}_b$ axis is taken according to the right hand rule. The position of the moving platform expressed in the reference system $\mathbf{S}_b$ is given as:

$$\mathbf{x} = \mathbf{b}_i + \mathbf{l}_i - \mathbf{Q}\mathbf{p}_i \qquad i = 1, 2, 3 \tag{1}$$

where: $\mathbf{b}_i = \left[\mathbf{O}_b\mathbf{B}_i\right]_{S_b}$; $\mathbf{p}_i = \left[\mathbf{O}_b\mathbf{P}_i\right]_{S_b}$; $\mathbf{l}_i = \left[\mathbf{B}_i\mathbf{P}_i\right]_{S_b}$; $\mathbf{x} = \left[\mathbf{O}_b\mathbf{O}_p\right]_{S_b}$ (2)

$\mathbf{Q}$ is the rotation matrix that takes $\mathbf{S}_p$ to $\mathbf{S}_b$. Since the 3-UPU manipulator has only translations, the rotation matrix $\mathbf{Q}$ is constant and can be considered as the identity. According to Eq. (1), the inverse kinematic model is given by:

$$l_i = \sqrt{\left(x-\left(r_b-r_p\right).\cos\left(\alpha_i\right)\right)^2+\left(y-\left(r_b-r_p\right).\sin\left(\alpha_i\right)\right)^2+z^2} \tag{3}$$

where l_i is the length of the i-th leg; α_i is the angular position of the i-th leg; *x, y* and *z* are the coordinates of the reference point of the platform O_p in $\mathbf{S}_b$.

2.2. Prediction of the position error

In this section, the method of intervals is applied to predict the upper and lower bounds of the orientation error of the manipulator. The interval operations are implemented using Matlab library INTLAB [6]. To reduce the overestimation, several contractors based on fixed-point iteration can be used. In this work, the "Krawczyk" contractor will be used. Let f be a function with variables (position error of the manipulator) and parameters given by vector $\mathbf{q}$. The variables and the parameters intervals are presented by [$\mathbf{x}$] and [$\mathbf{q}$] respectively. The solution of the system of equation $\mathbf{f}(\mathbf{q},\mathbf{x}) = 0$ is given as:

$$\sum(\mathbf{f},[\mathbf{q}],\tilde{\mathbf{x}}) = \{\mathbf{x}\in \mathrm{R}^n : \exists\ \mathbf{q}\in[\mathbf{q}], \mathbf{f}(\mathbf{q},\mathbf{x}) = 0\} \tag{4}$$

These solutions are closed to the nominal solution when the values of the parameters changes in its intervals. To avoid the overestimation, we will use the inverse kinematic model in its quadratic form:

$$l_i^2-\left(x-\left(r_b-r_p\right).\cos\left(\alpha_i\right)\right)^2-\left(y-\left(r_b-r_p\right).\sin\left(\alpha_i\right)\right)^2-z^2 \tag{5}$$

The system of equation given above takes the following form $\mathbf{f}(\mathbf{q},\mathbf{x}) = 0$.
Let $(\tilde{\mathbf{q}},\tilde{\mathbf{x}})$ be a nominal solution of the equation given above. The linearization of the function in the neighborhood of the nominal solution is given as:

$$\mathbf{f}(\tilde{\mathbf{q}},\tilde{\mathbf{x}}) + \mathbf{f}_{x}(\tilde{\mathbf{q}},\tilde{\mathbf{x}})(\mathbf{x}-\tilde{\mathbf{x}}) + \mathbf{f}_{q}(\tilde{\mathbf{q}},\tilde{\mathbf{x}})(\mathbf{q}-\tilde{\mathbf{q}}) = 0 \tag{6}$$

where $\mathbf{f}_x$ and $\mathbf{f}_q$ are the derivative of the function $\mathbf{f}$ with respect to $\mathbf{x}$ and $\mathbf{q}$, respectively. Thus, the solution of $\mathbf{f}(\mathbf{q},\mathbf{x}) = 0$ can be computed by the following equation:

$$\mathbf{x} = \tilde{\mathbf{x}} - \mathbf{C}.\mathbf{f}(\tilde{\mathbf{q}},\tilde{\mathbf{x}}) - (\mathbf{C}.\mathbf{f}_{x}(\mathbf{q},\mathbf{x}) - \mathbf{I})(\mathbf{x}-\tilde{\mathbf{x}}) - \mathbf{C}.\mathbf{f}_{q}(\mathbf{q},\tilde{\mathbf{x}})(\mathbf{q}-\tilde{\mathbf{q}}) \tag{7}$$

where C is a preconditioning nonsingular matrix [7, 3]
The mathematical concept of the "Krawczyk" operator can be written as:

$$\mathbf{K}_{\mathbf{f},[q]}([\mathbf{x}]) = \tilde{\mathbf{x}} - (\mathbf{C}.[\mathbf{X}] - \mathbf{I}).([\mathbf{x}] - \tilde{\mathbf{x}}) - [\mathbf{Y}] \tag{8}$$

where: $\mathbf{C} = \mathrm{mid}([\mathbf{X}])^{-1}$, $[\mathbf{A}] = [\frac{\mathrm{d}\mathbf{f}}{\mathrm{d}\mathbf{q}}]([\mathbf{q}],\tilde{\mathbf{x}})$, $[\mathbf{Y}] = \mathbf{C}.[\mathbf{f}](\tilde{\mathbf{q}},\tilde{\mathbf{x}}) + \mathbf{C}.[\mathbf{A}].([\mathbf{q}] - \tilde{\mathbf{q}})$ (9)

3 Multi-objective design optimization

A combined GA-"Krawczyk" algorithm will be used to determine the (DPNV) and their relative uncertainties that guarantee a good accuracy (E_p) with the maximal tolerance intervals of design parameters (IT). The population is evaluated by the interval linearization method: In fact, the position error of each design vector of the population is calculated with the "Krawczyk" algorithm (Fig.2).
Let the vector **q** regroups the (DPNV) and their uncertainties:

$$[\mathbf{q}] = [\tilde{r}_{\mathrm{b}}{}_{-\Delta r_{\mathrm{b\,min}}}^{+\Delta r_{\mathrm{b\,max}}}, \tilde{r}_{\mathrm{p}}{}_{-\Delta r_{\mathrm{p\,min}}}^{+\Delta r_{\mathrm{p\,max}}}, \tilde{\alpha}_1{}_{-\Delta\alpha_{1\max}}^{+\Delta\alpha_{1\min}}, \tilde{\alpha}_2{}_{-\Delta\alpha_{2\max}}^{+\Delta\alpha_{2\min}}, \tilde{\alpha}_3{}_{-\Delta\alpha_{3\max}}^{+\Delta\alpha_{3\min}}] \tag{10}$$

where $\tilde{q}_i$ is the nominal value of the corresponding design parameter and $\Delta q_{i\max}$ and $\Delta q_{i\min}$ are the upper and lower uncertainties values. The uncertainties of the design parameters nominal values are defined by an interval vector $[\mathbf{q}]$ and the position error of the manipulator by $[\mathbf{X}]$.

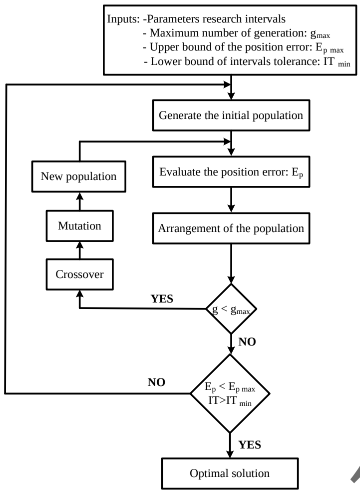


Fig. 2 Scheme of optimization process with genetic algorithm

The optimization problem can be formulated as:

$$\min \mathbf{F}(\mathbf{X}) = \min[f_j(\mathbf{X})]^{\mathrm{T}},\ (j=1..n) \tag{11}$$

Subjected to

$$\mathbf{g}(\mathbf{X}) < 0 \tag{12}$$

where T is the transpose operator, n is the number of objective functions, $\mathbf{X}$ is the vector of design variables, $\mathbf{F}(\mathbf{x})$ is the vector of objective functions and $\mathbf{g}(\mathbf{X})$ is the vector of constraint functions [11].

The goal of this work is to minimize the robot position error and maximize the design parameters tolerances' intervals simultaneously within a given workspace. The first objective function f_1 to be minimized corresponds to the platform position error, given by:

$$f_1 = \mathrm{Max}\left(\sqrt{E_{px}^2 + E_{py}^2 + E_{pz}^2}\right) \tag{13}$$

where: $E_{px} = \mathrm{Max}\left(|x_{min} - x|, |x_{Max} - x|\right)$, $E_{py} = \mathrm{Max}\left(|y_{min} - y|, |y_{Max} - y|\right)$,

$$E_{pz} = \mathrm{Max}\left(|z_{min} - z|, |z_{Max} - z|\right) \tag{14}$$

where $x_{\min}$, $y_{\min}$ and $z_{\min}$ are the lower bounds of the interval vector [**x**], $x_{\max}$, $y_{\max}$ and $z_{\max}$ are the upper bounds of the interval vector [**x**].
The second objective f_2 to be minimized corresponds to the inverse of normalized interval tolerance some. This objective function can be expressed as:

$$f_2 = \frac{1}{\frac{\Delta r_{\mathrm{b\,min}} + \Delta r_{\mathrm{b\,Max}}}{r_{\mathrm{b}}} + \frac{\Delta r_{\mathrm{p\,min}} + \Delta r_{\mathrm{p\,Max}}}{r_{\mathrm{p}}} + \frac{\Delta \alpha_{1\,\mathrm{min}} + \Delta \alpha_{1\,\mathrm{Max}}}{\alpha_1} + \frac{\Delta \alpha_{2\,\mathrm{min}} + \Delta \alpha_{2\,\mathrm{Max}}}{\alpha_2} + \frac{\Delta \alpha_{3\,\mathrm{min}} + \Delta \alpha_{3\,\mathrm{Max}}}{\alpha_3}} \quad (15)$$

To avoid the singularity related to the manipulator architecture, the following constraints have to be fulfilled: $r_{\mathrm{b}} > r_{\mathrm{p}}, \ \alpha_{\mathrm{i}} \neq \alpha_{\mathrm{j}}, \mathrm{i} \neq \mathrm{j}, \ \mathrm{i, j} = 1, 2, 3$ (16)

To avoid the problem of normalization of α_1, we choose a configuration where the angle first axe $\alpha_1 \neq 0$.

4 Case study

The desired workspace of the manipulator is a cube defined by:

$$-150 \leq x,y[\mathrm{mm}] \leq 150 \text{ and } 100 \leq z[\mathrm{mm}] \leq 400 \quad (17)$$

The uncertainties of the actuators lengths is chosen to be constant even the actuator lengths are variables: $\Delta l_{\mathrm{i\,min}} = \Delta l_{\mathrm{i\,max}} = 0.01\,\mathrm{mm}$. The bounds of the design parameters and their uncertainties are defined in Table1.

Table 1. Bounds of the design parameters and their uncertainties

Design parameters	*Bounds*	*Uncertainties of the parameters*	*Bounds*
r_b [mm]	[250 350]	Δr_b [mm]	[-0.3 0.3]
r_p [mm]	[20 80]	Δr_p [mm]	[-0.05 0.05]
α_1 [degree]	[80 100]	$\Delta\alpha_1$ [degree]	[-1 1]
α_2 [degree]	[210 230]	$\Delta\alpha_2$ [degree]	[-1 1]
α_3 [degree]	[320 340]	$\Delta\alpha_3$ [degree]	[-1 1]

A predefined function of genetic algorithm 'gamultiobj' is used to minimize the position error of the manipulator and to maximize the uncertainties of the design parameters. The parameters of the optimization algorithm are the population size $N_p = 200$, the maximum generation number $g_{max} = 100$, the crossover probability $P_c = 0.8$ and the mutation probability $P_m = 0.2$ .The Pareto front is used to represent the optimal solutions of the problem for the two competitive objective functions f_1 and f_2 . The main difficulty of a multi-objective problem is that the notion of optimal solution does not exist. The designer can simply accept the fact that one solution is preferable to another, so it is a question of finding satisfactory solutions. The Pareto front is given by fig 4.

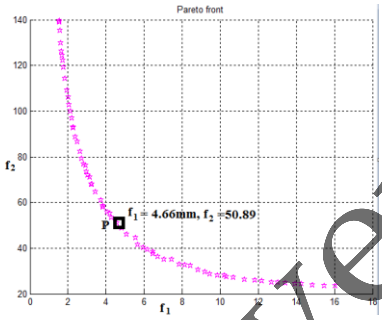
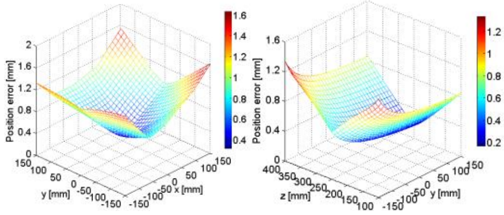


Fig. 4 The optimal design vectors

Hence, the designer can choose an optimal robust design vector from the Pareto front that suite his application. For example, if the designer chooses the solution P as presented in fig. 4, the allowed magnitude of the position error will be 4.66 mm. In this case, the chosen solution of design parameters nominal values and their uncertainties is given by:

$$[\mathbf{q}] = [271{,}28mm\,^{+0.17}_{-0.23},\ 53.59mm\,^{+0.03}_{-0.04},\ 87.64°\,^{+0.58}_{-0.57},\ 203.07°\,^{+0.24}_{-0.24},\ 330.0°\,^{+0.13}_{-0.2}] \quad (18)$$

For the chosen solution P, the distribution of the position error of the manipulator is presented in two sections of the workspace defined by y=0 and z=200. As it is shown in fig.5, the maximum position error does not exceed 4.66mm in the defined workspace.



a. z=200mm b. y=0

Fig.5 Distribution of the position error of the manipulator

5 Conclusion

In this paper, an hybrid GA-"Krawczyk" has been developed. The proposed method aims to determine the optimal design vector composed of the nominal design parameters and their uncertainties in order to minimize the position error of the manipulator. The proposed method is tested on the 3-UPU translational parallel manipulator and has proved its efficiency by determining the manipulator's design vector, in order to guarantee a better accuracy of the manipulator.

References

1. Boudreau, R. and Gosselin, C.M.: The Synthesis of Planar Parallel Manipulators with a Genetic Algorithm. ASME Journal of Mechanical Design, 121, pp. 533-537, (1999)
2. EL-Kribi, B. and Houidi, A. and Affi, Z. and Romdhane, L.: Application of multi-objective genetic algorithms to the mechatronic design of a four bar system with continuous and discrete variables. Mechanism and Machine Theory, 61, pp. 68–83 (2013)
3. Goldzstejn, A.: Sensitivity Analysis Using a Fixed Point Interval Iteration, HAL report number: hal-00339377 (2008)
4. Ping, J. and Hongtao, W.: Kinematics analysis of an offset 3-UPU translational parallel robotic manipulator. Robotics and Autonomous Systems, 42, pp.117–123 (2003)
5. Laribi, M.A. and Mlika, A. and Romdhane, L. and Zeghloul, S.: A combined genetic algorithm–fuzzy logic method (GA–FL)in mechanisms synthesis. Mechanism and Machine Theory, vol. 39, pp. 717–735 (2004)
6. Rump, S.: INTLAB – INTerval LABoratory, in : T. cendes (Ed.). Developments in reliable Computing, Kluwer Academic Publishers, Dordrecht, pp. 77-104 (1999)
7. Tannous, M. and Caro, S. and Goldsztejn, A.: Sensitivity analysis of parallel manipulators using an interval linearization method. Mechanism and Machine Theory, Elsevier,71, pp.93-114 (2014)
8. Tsai, L.W.and Joshi, S.: Kinematic and optimization of a spatial 3-UPU parallel manipulator. ASME Journal of Mechanical Design, 122, pp. 439-446 (1999)
9. Liu, X.-J and Wang, J. and Wu, C. and J. Kim: A new family of spatial 3DOF parallel manipulators with two translational and one rotational DOFs. Robotica, 27, pp. 241-247 (2009)
10. Zhen Huang, Z. and Chen, Z. and Liu, J. and Liu, S.: A 3DOF Rotational Parallel Manipulator Without Intersecting Axes. Mechanisms Robotics 3, doi:10.1115/1.4003848 (2011)
11. Ceccarelli, M. and Carbone, G. and Ottaviano, E.: Multi criteria optimum design of manipulators. Bulletin of the Polish Academy of Sciences, Technical Sciences, 53, pp. 9-18 (2005)