Holistic modular multilayer synthesis of planar linkages using parameterized mass properties

S. Heinrich[1] and M. Berger[2]

Dept. of Mechanical Engineering, Professorship Assembly and Handling Technologies, Chemnitz University of Technology, Reichenhainer Straße 70, 09126 Chemnitz, Germany
[1]*e-mail: stefan.heinrich@mb.tu-chemnitz.de*
[2]*e-mail: maik.berger@mb.tu-chemnitz.de*

Abstract. This paper shows that new opportunities and areas of development arise from using a module-based layer synthesis and description of mass properties combined with a domain spanning system simulation software. That allows for a holistic mechanism synthesis with a continuous analysis-synthesis-parameter-adjustment (ASPA). Hence, the user may perform both mechanism analysis and synthesis employing one single program interface with, what is also possible, an identification of the ideal mechanism in terms of dynamics during an optimization process.

Key words: modular multilayer synthesis, planar linkages, parameterized mass properties, rigid body dynamics

1 Introduction

For applying state of the art techniques to determine a dynamically favorable design of nonlinear mechanisms, it is necessary to know their characterizing parameters, such as structure related dimensions, mass, stiffness and damping matrices. In order to optimize a preferred nonlinear drive assembly by using high-performance programs (e.g. PTC Creo Elements/Mechanism, ESI ITI SimulationX or Altair HyperWorks) today's methods offer solutions like mass or power balancing. Despite their certain functions, these software tools sparely answer typical synthesis questions to fulfill arbitrary motion demands or dynamical boundary conditions. Therefore, a new method has been developed based on a local cross-linking of an analysis, synthesis and optimization to close the gap between the demands and already existing functions within a chosen software environment as shown in **Fig. 1**. In order to obtain this cross-linking, one main objective was to create a flexible method for everyday engineering practice. Hence, the principle of module based system engineering (as used in the software ITI SimulationX) was retrieved [1] which posed two main challenges. Firstly, it was necessary to develop a library of standardized synthesis modules that can easily be implemented within the aforementioned software environments and that provides the geometrical dimensions of a mechanism. The mathematical concept of this library is described in **Section 2**.

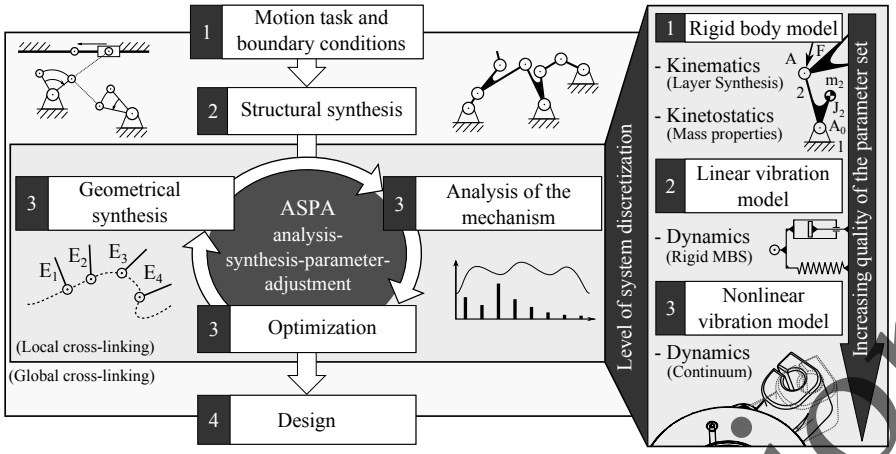

Fig. 1 Principle of an interactive analysis-synthesis-parameter-adjustment

A holistic mechanism synthesis with dynamical boundaries, like changing natural frequencies or reducing the foundation excitation, requires a proper description of all mass properties depending on the level of system discretization (established by Dresig [2]) shown in **Fig. 1**. Therefore, **Section 3** offers a suitable way to determine these parameters by approximating data sets gained from design studies of predefined solids or assemblies.

2 Modular geometrical synthesis

As mentioned before, it is necessary to create randomly connectable synthesis modules as shown in **Fig. 2**, that are able to communicate and interact with each other. Furthermore, they have to solve the synthesis and provide all relevant information for the computation of mass properties, explained in **Section 3**.

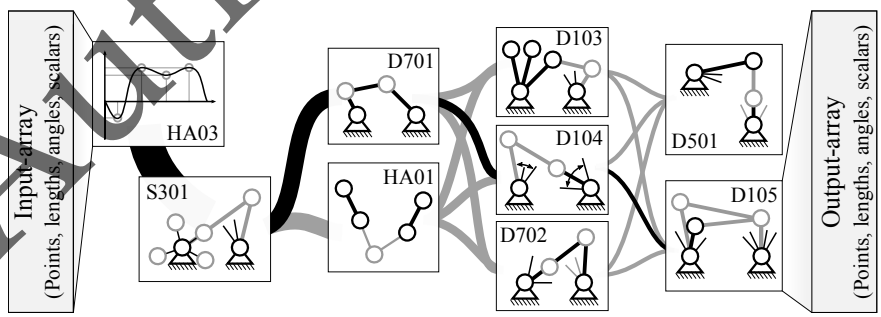

Fig. 2 Exchange of information within a connected modular synthesis

Therefore, each module uses point positions, lengths, angles and scalars to calculate the missing parameters describing every position of each point or plane. Each module returns a standardized data set including the missing positions, lengths, angles or scalars, such as the installation position of a joint point or other design factors. All modules are classified into five categories, shown in **Fig. 3**. The module V103, for example, is the third (03) variation of the four layer synthesis to compute a revolute joint within a moving frame of reference. This module can be used to find a coupler point on a moving link that is linked to a rocker. Furthermore, this module fulfills the boundary condition that in three out of four linkage positions the rocker does not rotate. **Section 4** contains further details on how to use the library.

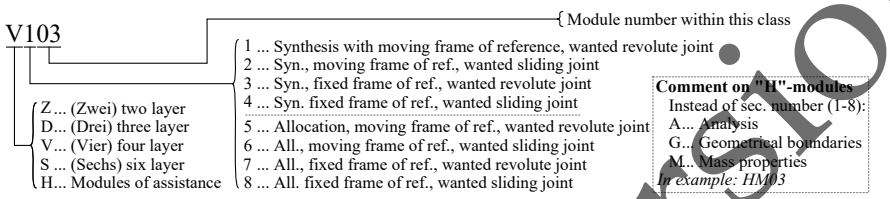

Fig. 3 Classification used within the new modular synthesis library

In order to implement such a library in any MBS or math software, it was necessary to find a suitable way to mathematically describe the solution for a given motion task. Accordingly, the basic approaches were vector analysis and complex numbers. During several studies it appeared that using complex indicators within the program language Modelica, which is the library language used in SimulationX, led to a significant increase of the effective calculation time. Despite this, it is a suitable way to compute the solution with mathematical software (for example Mathcad, Matlab or HyperMath). The following example of a three layer synthesis with a moving frame of reference explains the mathematical description. The module defines the revolute joint B in its three positions shown in **Fig. 4**.

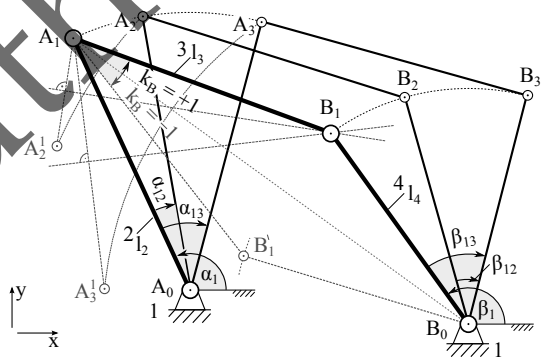

Fig. 4 Input and output of the synthesis module D103

The ingoing parameters of this module are the frame points A_0 and B_0 (given by x_{A0}, y_{A0}, x_{B0} and y_{B0}), the link length l_2, the angles α_1, α_{12}, α_{13}, β_{12} and β_{13}. A_0 is calculated with complex numbers and so is analogous to **Eq. (1)** B_0. The first step is to calculate the positions $A_{1,2,3}$ of the revolute joint point A according to **Eq. (2)** and **(3)**. In order to calculate the center point B_1, it is necessary to transform the positions of A back into position one of the frame of reference which is the rocker 4. Therefore **Eq. (4)** is used to rotate the points A_2 and A_3 around B_0.

$$A_0 = x_{A0} + y_{A0} \cdot i \qquad i = \sqrt{-1} \tag{1}$$

$$A_1 = A_0 + l_2 \cdot e^{(\alpha_1 \cdot i)} \tag{2}$$

$$A_n = A_0 + l_2 \cdot e^{([\alpha_1 + \alpha_{1n}] \cdot i)} \qquad n = [2,3] \tag{3}$$

$$A_n^1 = B_0 + (A_n - B_0) \cdot e^{(-\beta_{1n} \cdot i)} \qquad n = [2,3] \tag{4}$$

The next step is to calculate the position of B_1 as the intersection point using the two perpendicular bisectors on the sides $A_1 - A_2^1$ and $A_1 - A_3^1$ by employing **Eq. (5)**[1].

$$B_1 = \frac{(A_2^1 - A_3^1)|A_1|^2 + (A_3^1 - A_1)|A_2^1|^2 + (A_1 - A_2^1)|A_3^1|^2}{(A_1 - A_3^1)(A_2^1 - A_1) - \overline{(A_1 - A_2^1)}(A_3^1 - A_1)} \tag{5}$$

Calculating the positions B_2 and B_3 by rotating them around B_0 as shown in **Eq. (6)** is a faster way than applying the law of cosine due to the fact that the layer synthesis guarantees the adherence of all the three positions.

$$B_n = B_0 + (B_1 - B_0) \cdot e^{(\beta_{1n} \cdot i)} \qquad n = [2,3] \tag{6}$$

Finally, the missing module outputs are defined through **Eq. (7)** to **Eq. (9)**. **Eq. (7)** calculates β_1 (start angle) of the rocker. **Eq. (8)** delivers the rocker and link length and **Eq. (9)** is used to define whether the installation position factor k_B is positive $(+1)$ or negative (-1). This last factor is an important information for the module based analysis of the synthesized linkage according to VDI 2729 [3].

$$\beta_1 = Im(ln(B_1 - B_0)) \tag{7}$$

$$l_3 = |B_1 - A_1| \quad l_4 = |B_1 - B_0| \tag{8}$$

$$k_B = \begin{cases} 1, & \text{if } Im\left(ln\left[\frac{B_1 - A_1}{B_0 - A_1}\right]\right) > 0 \\ -1, & \text{otherwise} \end{cases} \tag{9}$$

As a result of these calculations, the module D103 returns the exact positions of $A_{1,2,3}$ and $B_{1,2,3}$, the angle β_1, the link lengths l_3, l_4 and the installation position factor k_B. That information allows the modules to be included in a complex mechanism

[1] The vinculum within the denominator is used for the conjugated complex vector

simulation. Furthermore, it is now possible to compute mass parameters with those synthesis solutions to achieve further analysis.

3 Parameterized mass properties

Anticipating nearly exact mass properties during a layer synthesis to determine the dynamical behavior of a drive assembly combined with its optimization leads to the issue dealt within this section. There are several different ways to compute mass properties from solids with or without the CAD environment. Typically, a CAD software continuously calculates the moment of inertia $\underline{\underline{J}}$ according to **Eq. (10)** by knowing all the relevant model parameters (constant density ρ, the position vectors $\underline{r}$) and especially the bounds of integration due to the design process. Bearing that in mind, it would be necessary to create CAD features within typical MBS or math software if it was necessary to compute mass properties in the exact same manner.

$$\underline{\underline{J}} = \rho \int_V \left((\underline{r} \cdot \underline{r}) \underline{\underline{I}} - \underline{r} \otimes \underline{r} \right) dV \qquad \underline{\underline{I}} \ldots \text{identity tensor} \tag{10}$$

However, there are five ways to describe the unknown mass parameters which can easily be implemented:

1. Co-simulation with parametric models and a continuous exchange of parameters
2. Parameter identification based on a CAD design study
3. Manipulation of triangulated solids from STL files
4. Determination of mass properties by using primitive solids
5. Using discrete mass points for a compensatory moment of inertia

Referring to number 2, links in non linear mechanisms can be standardized by typically used design strategies shown in **Fig. 5**. Therefore, it is suitable to create a data set through a design study (VC) of predefined solids or assemblies within the CAD environment.

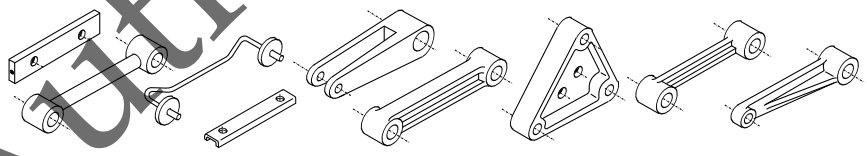

Fig. 5 Typical designs of linkages within mechanisms drawn by Volmer [4]

Such a data set contains the moment of inertia (J_S) about the polar axis, the position of the center of gravity (S) and the mass (m) of the solid. Furthermore, it contains all varied parameters, as for example a length or an angle. It occurs that using the Gauss method of least squares is also a practical way to approximate those

mass parameters through a p-dimensional polynomial approach function. A ternary link, for example, can be described by three variable parameters: its two lengths l_{AB}, l_{AC} and the angle γ_{BAC}. This leads to a one, two or three dimensional polynomial approach function depending on the number of used variables. **Eq. (11)** represents such a p-dimensional polynomial approach function of the order n that can be used for the Gauss method.

$$P_n(x_1,\ldots,x_p) = \sum_{n_1+\ldots+n_p<n} b_{n_1,\ldots,n_p} \cdot x_1^{n_1} \cdots x_p^{n_p} \tag{11}$$

Therefore, the minimization problem for a data set with m entries per dimension p is defined by **Eq. (12)**. In this equation, $\mathbf{b}$ represents the coefficient vector, $\mathbf{x_i}$ the i^{th} p-dimensional variable vector from the data set and $\mathbf{y_i}$ the attached mass parameter.

$$\min_{\mathbf{b}} \sum_{i=1}^{m} \Big(f(\mathbf{x_i},\mathbf{b}) - y_i \Big)^2 \tag{12}$$

Solving **Eq. (12)** leads to the polynomial coefficients of a p-dimensional function, in this case an area function as shown in **Fig. 6**. In this example, the two parameters γ_{BAC} and l_{AC} of the ternary link *A-B-C* were varied. The polynomial regression led to one area function for each mass parameter vector from the data set.

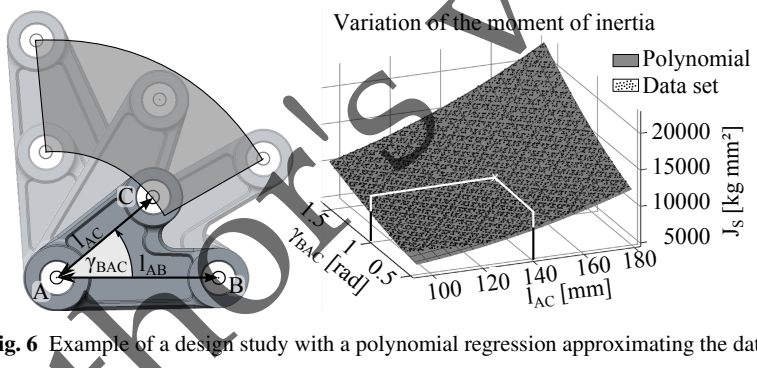

Fig. 6 Example of a design study with a polynomial regression approximating the data

Applying these polynomial functions to describe all the mass parameters gives the opportunity to investigate dynamical characteristics through a modal analysis or the distribution of the total energy of the mechanism.

4 Example of a parameterized multilayer synthesis

This section deals with an example of a synthesis task based on a mechanism promoted by Socha in 1967 [5] that consists of 8 links. In **Fig. 7** only the lower, and for this section relevant part of the complex mechanism, is shown. With the help of the

introduced modular synthesis and description of link mass parameters it is possible to calculate the system behavior and study sensitivities by varying free parameters. Usually, a geometrical layer synthesis leads to a variety of possible link lengths and angles, all fulfilling given motion demands. From such a variety of solutions the designer has to choose his favorable parameter set according to recommendations from the literature. Unfortunately, those are usually based on geometrical parameters like the transmission angle (see **Fig. 7**) which is not enough for meeting stronger boundary conditions on its dynamical behavior.

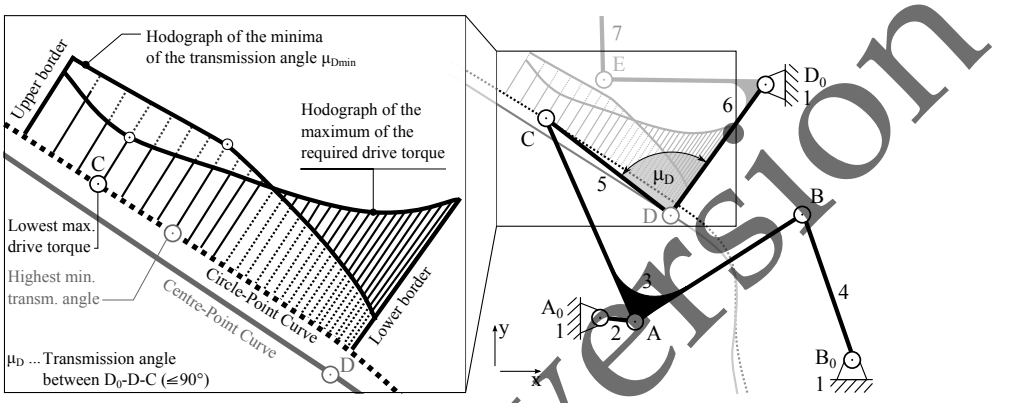

Fig. 7 Section of a coupler mechanism with a position variation on the Burmester curves [6]

A variation of the position of C on the circle-point curve [2] during the layer synthesis directly affects the required drive torque based on the mass and energy distribution in the mechanism. By having a closer look on the drive torque curves from **Fig. 8** it occurs that even within a small interval this variation has a huge impact on the maximum torque. The upper and lower border are defined through the given boundary conditions, that can be looked up in [6].

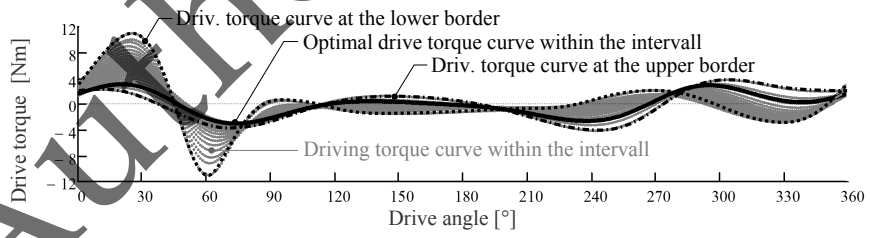

Fig. 8 Torque curves resulting from a parameter study within the given interval in **Fig. 7**

Comparing the hodograph of the required maximum drive torque with the hodograph of the transmission angle from **Fig. 8** shows that a dynamical optimum not

[2] The circle- and center-point curves are also known as Burmester curves in a four layer synthesis.

necessarily has to be a geometrical optimum. The optimal drive torque curve can be reached by choosing a position on the circle-point curve close to the geometrical optimum. Further explanations on this example can be found in [6].

5 Conclusion

Determining a dynamically favorable design of a nonlinear mechanism usually leads to an iterative method based on experience and some imprecise recommendations from the literature. Thus, the classical geometrical layer synthesis often cannot meet the given dynamic boundaries. Referring to **Fig. 7**, a geometrical based optimization of a linkage could result in a dynamical disadvantageous mechanism. Trying to close the gap between dynamics and classical layer synthesis led to two challenges. The first objective was to develop a modular synthesis library based on algorithms that easily can be implemented in various MBS software. The second and more challenging objective was to find a suitable way to determine mass properties that are necessary for proper modeling. The shown approach of using design studies combined with a polynomial regression provides the opportunity to investigate the dynamical behavior during the synthesis. Based on an exact description of the mechanism mass parameters, it is now possible to improve the mechanism model. Especially when it comes to further studies on the mechanical system behavior, it is inevitable to know the exact moment of inertia (J_S) about the polar axis, the position of the center of gravity (S) and the mass (m). Further investigations on this issue will answer the following questions:

1. How can a variation of a design parameter from the layer synthesis positively influence the interchange of energy between the links, regarding the total amount of energy?
2. How does the variation of design parameters interfere with the response to parametric excitation, based on bearing play or stiffness?

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