

On the Motion/Force Transmissibility and Constrainability of Delta Parallel Robots

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Abstract. The motion and force transmission is highly important for the analysis and design of parallel manipulators. Recent advances in research have led to generally applicable formulations for transmission indices based on the notion of power coefficient. Analyses of limited-dof parallel manipulators however require separate consideration of constraint characteristics. Conversely, the design parameters of parallel manipulators are highly coupled. Thus, such separation may distort the performance evaluation and optimization of parallel manipulators. In this context, indices based on pressure angles of fully parallel manipulators are revisited and applied to the performance evaluation of the Delta robot, one of the lower-dof parallel robots. The resulting index is physically appropriate and allows for simultaneous assessment of both, the motion and force transmission and the constraint characteristics.

Key words: Transmission Indices, Pressure Angle, Power Coefficient, Constraint Singularity, Delta Parallel Robot.

1 Introduction

The geometries of parallel manipulators can be optimized such that specified workspace requirements are met. Manipulators that were optimized by workspace and occupied space requirements only may however suffer from poor kinematic and dynamic characteristics. Thus, kinematic and dynamic performance measures are commonly taken into account for the optimization of manipulators. The main kinematic concepts for performance measurement are the concepts of condition number, manipulability, and motion/force transmissibility and constrainability [1].

Both measures, the condition number as well as the manipulability, are based on the characteristics of the Jacobian. The condition number [2] is a local measure of the Jacobian-induced distortion of the motion and force transmission from the active joint to the end-effector space. The product of the singular values of the Jacobian matrix corresponds to the volume of the so-called manipulability ellipsoid [3]. However, information on the directionality get lost. In addition, for transla-

tional and rotational dof of the moving platform, the Jacobian matrix contains inhomogeneous units and further modification, e.g. normalization or separate analyses of position and orientation [4], is required. Moreover, for limited-dof manipulators the input-output Jacobian may not be sufficient to predict all possible singularities [5]. Finally, Jacobian-based indices are frame-dependent. As a result, the values of the indices vary with the choice of coordinates [6]. To overcome the aforementioned problems, the performance of parallel manipulators can alternatively be assessed analyzing the quality of transmissibility and constrainability.

Analyses of transmissibility date back to Alt [7] proposing the transmission angle in planar mechanisms. Following the theory of screws [8], the first transmission indices of spatial manipulators were proposed by Yuan et al. [9] using the virtual coefficient between the transmission wrench screw (TWS) and the output twist screws (OTS). Sutherland and Roth [10] normalized the initial approach. Shimojima et al. [11] proposed a unique definition of TWS, which is dependent on the output link's load condition. Further generalizations were proposed by Tsai and Lee [12] taking into account a generalized transmission wrench screw (GTWS) and the related virtual coefficients to the input and output screw. Later Chen and Angeles [13] proposed the generalized transmission index (GTI). The three approaches can be distinguished by the different definitions of the maximum value of the virtual coefficient (as used for normalization).

Takeda and Funabashi [14] proposed a transmission index (TI) taking into account the virtual power transmitted from the input links to the output link. In their approach, single-dof mechanisms are generated by fixing all input links except one and analyzed in respect of the resulting pressure angles at the connection between input and output link. The approach is only feasible for TWS with a zero pitch (i.e. a transmission force line) and thus represents a special case of the GTI. In other words, in order to define the pressure angle in a simple definition, the TWS can be represented at a (spherical) joint where no moment is applied as constraint. This approach was extended to cable driven parallel mechanism [15] and spherical parallel mechanism [16]. Briot et al. [17] investigated the determination of the maximum reachable workspace of planar parallel manipulators based on the transmission angle and the position of the instantaneous center of rotation.

Based on the concept of virtual coefficient and following Takeda's approach of fixing all inputs except one, Wang et al. [18] proposed a general procedure for non-redundant spatial parallel manipulators including new transmission indices based on the input transmission index (ITI) and output transmission index (OTI), where the normalized virtual coefficient is called power coefficient. Additionally, the minimum of all indices are defined as local transmission index (LTI). The concept was extended for redundant and/or overconstrained parallel manipulators [6, 19]. Further indices are e.g. the global transmission index (GTI) ensuring good performance throughout the entire workspace of a manipulator and the good-transmission workspace (GTW) defined by a minimum value for the LTI [20]. In fact, the proposed indices are able to detect a manipulator's closeness to actuation (transmission) singularities, but cannot be applied to measure the closeness to con-

straint singularities [21, 22]. Thus, constraint transmission indices (CTI) were developed as shown in [21, 23] and further refined and discussed extensively in [24]. At the same time, Liu et al. [25] proposed a novel approach for the derivation of the maximum value of the virtual coefficient.

In the present paper, transmission indices based on the approaches of both the pressure angles and the power coefficient for the Delta robot, one of the lower-dof parallel robots, are formulated. Assessments are given to support the kinematic design of the Delta robot with high transmission and constraint capability.

2 Transmissibility of Delta Parallel Robots

The Delta robot is one of the best known and most widely spread parallel robots in academia and industry [26]. The output link or moving platform of the Delta robot is restricted to purely translational dof. Usually, the architecture is represented by three symmetric kinematic chains of the type $\underline{R}(\underline{SS})_2$. Accordingly, the parallelogram contains four spherical joints and four links pairwise of the same length. With this, the connecting rods only need to transmit axial forces allowing for light-weight design. Fig. 1 shows the schematic representation of the Delta robot and the related kinematic relations. The vector $r_{F,i}$ denotes the position of the revolute joint of the i -th kinematic chain on a circle with radius r_F . Similarly, the attachment point of the parallelogram on the output link is denoted by $r_{P,i}$. The distance D_s between the spherical joints is the same for all joint pairs. The vectors $l_{PL,i}$ and $l_{DL,i}$ point along the i -th proximal and distal links, respectively. The workspace prescribed for the following analyses is represented by a cylindrical base (with radius D_1 and height Z_1) and a conical portion (with radius D_2 and height Z_2) adjacent to it. The centre of their connecting surface determines the relative position $P_0 = [0, 0, Z_0]$ of the workspace to the origin of frame O. Table 1 summarizes the related parameters. The fundamentals are presumed to be known.

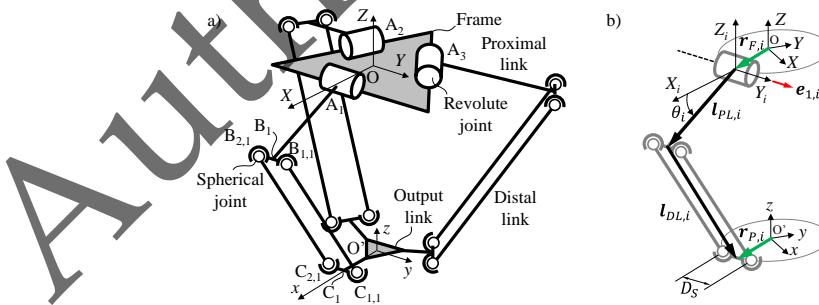


Fig. 1 Schematic representation and definition of kinematic parameters

Table 1. Parameters: Delta robot and prescribed workspace

Parameter	$r_{F,i}$	$r_{P,i}$	D_s	$l_{PL,i}$	$l_{DL,i}$	D_1	D_2	Z_1	Z_2	Z_0
Value [m]	0.20	0.05	0.10	0.40	0.80	1.00	0.80	0.25	0.05	-0.80

2.1 Transmission indices based on pressure angles

The two connecting rods j within a chain i solely transmit axial forces denoted by the unit vector $\hat{\mathbf{f}}_{j,i}$. Hence, the direction of these forces is given by the vector $\mathbf{l}_{DL,i}$ (Fig. 1b). The angle between the velocity $\mathbf{v}_{B,i}$ of the spherical joint $B_{j,i}$ and the direction of the force transmitted to the output link along the distal link can be interpreted as pressure angle γ_i of the input transmission. Its cosine value is thus

$$\lambda_{i,PA} = \cos(\gamma_i) = \mathbf{v}_{B,i}^T \hat{\mathbf{f}}_{j,i} / \|\mathbf{v}_{B,i}\| \quad (1)$$

Accordingly, the best transmission occurs when the directions of velocity and force coincides. The input transmission index ITI is given as the minimum of the absolute pressure angles' cosine among all three kinematic chains, i.e.

$$ITI = \min(|\lambda_{i,PA}|) \quad \forall i = \{1,2,3\} \quad (2)$$

In respect of the output transmission, a transmission wrench screw (TWS) can be introduced at each of the six spherical joints $C_{j,i}$ where no moment is applied as constraint [14, 27]. With this a simple definition of an output-related pressure angle is obtained. The TWS of rod (j, i) with respect to $C_{k,m}$ are then given by

$$\hat{\$}_{TWS,j,i} = \begin{bmatrix} \hat{\mathbf{f}}_{j,i} \\ (\mathbf{r}_{P,j,i} - \mathbf{r}_{P,k,m}) \times \hat{\mathbf{f}}_{j,i} \end{bmatrix} \quad (3)$$

Imagine the virtual motion of a single-dof mechanism by removing one of the six connecting rods k of a chain m (Fig. 2b, $k = 1$) [14]. The instantaneous motion of the output link is the given by

$${}^{(k,m)}\$_{OTS} = \begin{bmatrix} {}^{(k,m)}\boldsymbol{\omega} \\ {}^{(k,m)}\mathbf{v}_{j,i} \end{bmatrix} \quad (4)$$

with ${}^{(k,m)}\boldsymbol{\omega}$ as angular velocity and ${}^{(k,m)}\mathbf{v}_{j,i}$ as translational velocity at joint $C_{j,i}$ of the output link. Among $\hat{\$}_{TWS,j,i}$, all except $\hat{\$}_{TWS,k,m}$ are constraint wrenches applying no work to the output link [18]. Then, the virtual power can be derived as

$$\delta W_{j,i} = \hat{\$}_{TWS,j,i} \circ {}^{(k,m)}\$_{OTS} = \hat{\$}_{TWS,j,i} \circ \begin{bmatrix} {}^{(k,m)}\boldsymbol{\omega} \\ {}^{(k,m)}\mathbf{v}_{j,i} \end{bmatrix} = 0 \quad (5)$$

where one entry of ${}^{(k,m)}\$_{OTS}$ can be chosen arbitrarily. Finally, the instantaneous velocity ${}^{(k,m)}\mathbf{v}_{j,i}$ at joint $C_{k,m}$ can be extracted to compute the pressure angle $\alpha_{k,m}$ of the output transmission. The angle can be physically interpreted as pressure angle at the respective connection point to the output link (Fig. 2b). Its cosine value is given by

$$\eta_{k,m,PA} = \cos(\alpha_{k,m}) = {}^{(k,m)}\mathbf{v}_{k,m}^T \hat{\mathbf{f}}_{k,m} / \|\mathbf{v}_{k,m}\| \quad (6)$$

The output transmission index can then be derived as

$$OTI_{PA,6} = \min(|\eta_{k,m,PA}|) \quad \forall m = \{1,2,3\}, \forall k = \{1,2\} \quad (7)$$

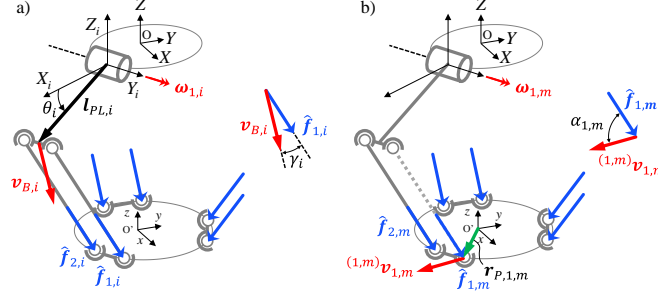


Fig. 2 Pressure angles of input and output transmission (with six supporting links)

2.2 Transmission indices based on power coefficients

Alternative approaches are based on the notion of power coefficient. In general, the orthogonal product of a wrench and twist screw ($\hat{\$}_{WS}$ and $\hat{\$}_{TS}$) related to a body is called virtual coefficient and can be interpreted as instantaneous power caused by the wrench acting on the moving body [24]. The higher the virtual coefficient, the better is the kinematic performance or the less wrench is required to transmit power [25]. The power coefficient is the normalized virtual coefficient

$$\rho = |\hat{\$}_{WS} \circ \hat{\$}_{TS}| / |\hat{\$}_{WS} \circ \hat{\$}_{TS}|_{max} \quad (8)$$

Following this definition, the input transmission index corresponds to Eq. (1).

For the output transmission index, the unit output twist screw (OTS) is related to the TWS. The general derivation in [25], and in particular the derivation of the maximum characteristic length, can be simplified for the Delta robot. The axis of the TWS passes through the spherical joint for any configuration. More importantly, presuming that the output link performs translational motion only, the angular velocity of the OTS is set to zero (disregarding potential constraint singularities). Then, for infinite pitch screws (pure translation), the maximum virtual coefficient is simply given by the maximum value of the dot product of the wrench and twist axes, which is one in this case [13].

Delta-related analyses of the output transmission based on power coefficients can, for example, be found for the four-legged 4-dof-variants X4 [28] and Ragnar robot [29]. Presuming three supporting links and translational motion of the output only, the system of Eqs. (5) can be solved releasing one input link while blocking the other two. For instance, if the first chain is removed the output link might move in the direction

$$\hat{\$}_{OTS,1} = \begin{bmatrix} \mathbf{0}_3 \\ \mathbf{v}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{0}_3 \\ \hat{\mathbf{f}}_3 \times \hat{\mathbf{f}}_2 / \|\hat{\mathbf{f}}_3 \times \hat{\mathbf{f}}_2\| \end{bmatrix} \quad (9)$$

Then, an alternative formulation for the output transmission index is given as

$$\eta_{1,PC,3} = |\hat{\$}_{TWS,1} \circ \hat{\$}_{OTS,1}| = |\hat{\mathbf{f}}_1^T (\hat{\mathbf{f}}_3 \times \hat{\mathbf{f}}_2)| / \|\hat{\mathbf{f}}_3 \times \hat{\mathbf{f}}_2\| \quad (10)$$

which, since the maximum virtual coefficient is one, corresponds to the orthogonal product of wrench and twist screw. The same applies for the second and third chain. As mentioned before, the orientation of the output link cannot be kept constant if one complete chain is removed since one constraint moment is removed as well. Compared to the OTI taking into account all six supporting links, cf. Eq. (3), Eq. (10) disregards the constraint moment and thus prevents a physically appropriate definition as pressure angle. Still, in recent Delta-related studies (e.g. [21], [30], [31]) the index is introduced as pressure angle among the three supporting links where three unit forces \hat{f}_i apply to the output link from the three chains. Then, for the 3-dof variant, the absolute value of the cosine of the pressure angle among the links corresponds to Eq. (10). Inevitably, the constraint singularities must be tackled separately using constraint transmission indices as shown in [23].

3 Results and Discussion

The transmission characteristics are analyzed based on the parameters as shown in Table 1. The input transmission characteristics (ITI) are unequivocal, whereas two distinct approaches are found for the output transmission. These are (A1) based on physically appropriate pressure angles of six supporting links covering constraint singularities ($OTI_{PA,6}$) and (A2) based on the power coefficient presuming translational (virtual) motion of the output link with three supporting links ($OTI_{PC,3}$). Fig. 3a) displays the distributions of ITI for the symmetry planes of the prescribed workspace and the $Z = Z_0$ -plane. Fig. 3b) shows the distribution of the OTI for $Z = -0.24$ and unrestricted swing angles. Regions where OTI become less than 0.1 are highlighted in red. Here, based on the relationship between the output pose error and the transmission index (TI) for the 6-SPS mechanism in [32], the threshold value to identify the neighborhoods of singularity is set to 0.1.

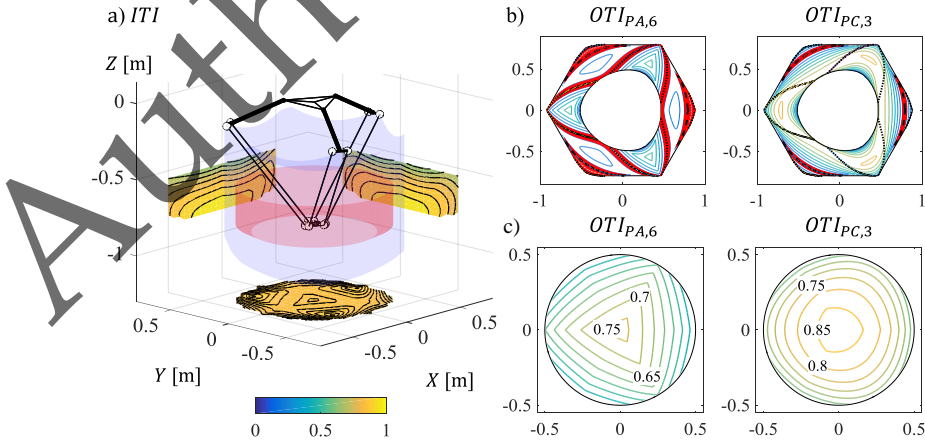


Fig. 3 Distribution of the input and two alternative output transmission indices

For (A1), the $OTI_{PA,6}$ are close to zero in neighborhoods where actuation (dashed line) or constraint (dotted line) singularities occur. Accordingly, (A2) fails to detect the internal (constraint) singularities. However, minimal values ($OTI_{PC,3} \leq 0.1$) correspond to singular point-curves of actuation. Thus, for thorough analyses with (A2) separate investigations of constraint transmission indices are unavoidable. Then, the same singular point-curves can be obtained as for $OTI_{PA,6}$. Nonetheless, in contrast to $OTI_{PA,6}$, the definition of OTI and CTI is based on virtual situations which are not physically appropriate. For instance, for the CTI , the relation between the constraint wrench moment and the (virtual) rotational motion of the output link is evaluated. In practice, such situation does not exist. Moreover, difficulties may arise for the determination of a unique index, which may be the minimum or the product of OTI and CTI including weightings. Accordingly, the distributions of the OTI on the $Z = Z_0$ -plane within the prescribed workspace are different (Fig. 3c). In summary, (A1) seems to be the most appropriate approach for the analyses and understanding of the motion/force transmissibility and constrainability of Delta parallel robots.

4 Conclusions

This study demonstrated different approaches to assess the transmission and constraint capabilities of Delta robots. Recent advances based on the power coefficient can be used for the analyses with three supporting legs. However, for lower-dof parallel manipulators, constraint characteristics must be assessed together with actuation (output transmission) characteristics. Therefore, an alternative approach based on pressure angles is introduced. The resulting index is physically meaningful and simultaneously takes into account actuation as well as constraint characteristics. Moreover, using this index, the distance between the spherical joints in a parallelogram of the distal links can be considered as an additional design parameter in future kinematic optimization of Delta parallel robots. Future work includes the generalization of the proposed approach based on the pressure angles for the evaluation of the transmission and constraint characteristics of lower-dof parallel robots.

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