Robust Design Methodology of Topologically optimized components under the effect of uncertainties

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Abstract

In the present work, a robust design methodology is presented for topologically optimized components. An integrated methodology combining design of experiment and reliability based topology optimization is proposed to capture the performance of optimized components in realistic environment including various uncertainties. In the present work, compliance, output displacement and the maximum von-Mises stress values are considered as performance functions. Volume fraction, force and aspect ratio are set as design-factors. The uncertainties of design factors are incorporated in the design using reliability method. The uncertainties of non-controllable factors are simulated by creating random field of material properties. Considering uncertainties, the performance of the topology optimization problem is simulated for a space of design factors. The simulated results are analyzed using statistical tools such as, analysis of mean. This technique helps to identify statistical significance and the effect on the performance variations. The proposed methodology is illustrated on a Force inverter. This analysis provides a design methodology in a realistic environment that helps in achieving targeted performance and robust design.

Key Words: Topology optimization, Robust design, Uncertainty, RBTO, Design of Experiments, Reliability

1 Introduction

Topology optimization is a useful tool for minimization of structural and machine components [1,2]. This is done taking into consideration a set of boundary conditions with an objective to maximize the performance of the component.

The performance of a topologically optimized structure is characterized by the mechanical advantage, output displacement and the maximum stress developed. These factors depend on parameters like input force, volume fraction, elasticity of the workpiece, dimension of the material etc. However, in reality there is a considerable amount of uncertainty involved. Consequently, this would affect the performance of the structure. Therefore to deal with this problem, Reliability based Topology Optimization (RBTO) is utilized.

In this paper we consider input force, volume fraction and aspect ratio as the input factors. These are controllable input factors, since they can be modified after each experiment. The elasticity of the material is an non-controllable input factor. The general approach followed in RBTO is to consider the uncertainty in design factors and to generate the performance values for the worst case scenario. In this approach however, it is difficult to take into account the uncertainties in the non-controllable factors. In addition, it is difficult to conduct the performance analysis of the experiment for all values of the design factors. Hence, to simplify these issues we integrate the method of Design of Experiment (DOE) with Reliability based Topology Optimization (RBTO) [6,7,14,16]. Using DOE helps us to establish the relation between the performance function values and the input factors. It also enables us to perform the simulations in the created design factor domain systematically, thereby reducing the number of simulations. The performance function values are analyzed using statistical techniques like Analysis of Mean (ANOM), Analysis of Variance (ANOVA) and Signal to Noise Ratio (SNR). In the present work, all the above-mentioned techniques have been carried out for a force inverter.

The manuscript is organized in following manner. The descriptions of DOE and RBTO are presented in Sections 2 and 3 respectively. The overall methodology of simulation is given in Section 4. The analyses of the simulated results are discussed in Section 5. The performance values are verified for the structure in COMSOL and finally a conclusion is drawn.

2 Design of Experiments

Design of Experiments (DOE) is a systematic process to frame efficient experiments [10]. The objective of the experiments would be to analyze the effects of several factors on the response or performance of a product or a process. Factors are the parameters that affect the performance of the process or product. These are initially set by the designer while commencing his study. Using the normal experimental process, the number of experiments may be very large to generate the factor-response relation in a generalized way. Here, the DOE offers a scientific way to choose numbers and type of experiment to reduce the cost of experiment without any loss in efficiency. This is achieved by merging several design factors in one study in spite of conducting separate study of each factor. In this way, number of experiments decreases and detailed understating of the product performance is got. In addition to the above, DOE approach helps in steering performance in a desired direction. Therefore, statistically significant factors can easily be identified, and the treatment combinations that have reduced variations in the performance can be identified.

We have chosen input force, volume fraction and aspect ratio as the input parameters. Their uncertainties are selected based on the literature available [11]. Since we have chosen three factors there are a total of 27 combinations possible. From these values we have selected 9 combinations to perform the simulation based on Taguchi method [12]. The uncertainties in the non-controllable factors are also included while carrying out the simulation.

The overall framework of a DOE is represented in Fig. 1. Here, the overall experiment is designed by DOE approach and simulations of performance functions are carried by RBTO method [4,13]. The responses (performance) corresponding to this combination will be the outcome of this experiment. Using the available ANOM, ANOVA, SNR and Response Surface method the analysis of the simulated result can be performed. The detailed discussions on each of these techniques, with the actual results are given in subsequent sections. As mentioned earlier, the approach of DOE as integrated with the RBTO method is discussed in the next section.

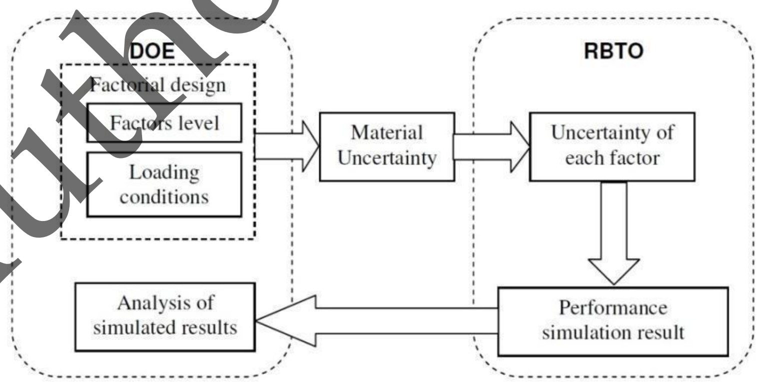


Fig. 1 Framework of integrated DOE and RBTO approach

3 Design of Experiments Integrated to Reliability Based Topology Optimization

In the present work, DOE method is integrated with RBTO. The overall experiment is designed by DOE approach and simulations of performance functions carried by RBTO method. The responses (performance) corresponding to this combination will be the outcome of the experiment (Fig. 1).

The probability of failure of the design [3, 15] is included through the extra probability constant, typical of any RBTO problem. Different types of optimal topologies can be generated based on the type of RBTO approach [4,13]. However, we follow the approach proposed by Kharmanda et al. [8,9], since this method minimizes the computation time involved in solving RBTO problems. Thus the present investigation of simulating the compliance and deflection values of reliable optimal topology, employing the Kharmanda [8,9] approach would involve the following steps.

The design variables are represented by means of variables *x* and the random variables are represented as *y*. The reliability index (β) is introduced with a normalized vector ***U*** that defines the relation between *x* and *y* [5]. The variables *x* and *y* can be related to each other by the normalized vector ***U***. The vector ***U*** is used to relate random values with mean and also random values with spread (or standard deviation).

The spread *S* is related to the standard deviation σ as S= 6σ

The next step is to evaluate the reliability index β. This is done by solving a constrained optimization problem shown below. The solution of this is called the design point.

$$min: \beta = \left(U_g^T U_g\right)^{\frac{1}{2}}$$
$$subject\ to: \beta \geq \beta^*$$

Now the reliability index when used with the normalized vector ***U***, defines the change in the value of the design variables. Using these values the topology optimization problem is carried out, with an additional constraint to satisfy a target reliability index β.

4 Methodology

In the present work as explained in Sections 2 and 3, we follow the procedure given below to analyze the performance of the topology optimized component with respect to different levels of factors, including uncertainties.

The set of controllable factors for the problem in our study are applied force, volume fraction and aspect ratio. Using the reliability index and spread values the uncertainties are calculated. Taking these uncertainties into consideration the optimal topology is generated and the performance functions namely compliance, output displacement and von- Mises stress values are determined. Compliance is calculated taking the ratio of the reaction force at the output point to the force exerted at the input. Von Mises stress is calculated by taking the deflection in each direction and substituting it in Eq. (1), where 1, 2 3 are the stresses in individual directions.

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_1\sigma_3 - \sigma_2\sigma_3} \qquad (1)$$

The above mentioned steps are carried out taking the problem of a force inverter and simulated using MATLAB.

4.1 Reliability Based Topology Optimization for a force inverter

To illustrate this work we use a force inverter. The design domain for a force inverter is shown in fig. 2. Force inverter is an example of a compliant mechanism. For certain applications in MEMS there is a need to convert contraction forces to expansion forces. We us a force inverter for this.

Fig. 2 Design domain with load scheme for force inverter

The goal of the optimization problem is to maximize the mechanical advantage so that the energy stored as strain energy is minimum. Strain energy uses a portion of the input energy hence the output energy reduces making the force inverter less efficient. A spring is attached at the output and the input port to simulate the resistance offered by the workpiece. The optimal topology for the force inverter is to be generated for a required material volume so that the mechanical advantage is maximum.
As shown in the figure the dimensions of the design domain is 300 μm× 300μm and the thickness is 10μm. We take the material of the mechanism as silicon which has Young’s modulus 160GPa and poison ratio 0.22. An input force is applied at the middle of the left edge and an output force is obtained at the middle of the right edge. The left top and the left bottom corners are fixed. Now for a given volume constraint we have to maximize the mechanical advantage. Let us say we require only 30% of the total volume of the domain. We assume the spring constant (elasticity) of the workpiece as 30N/μm. The maximum allowed displacement at the input end is set as 2 μm. The values of the design factors which we employ to generate the optimized topology and to calculate the value of performance functions are given in Table.

Table 1. Level of factors

Force (Newton)	*Volume Fraction*	*Number of elements in the x-direction*
0.001	0.30	30
0.001	0.35	32
0.001	0.40	34
0.002	0.30	34
0.002	0.35	30
0.002	0.40	32
0.003	0.30	32
0.003	0.35	34
0.003	0.40	30

5 Analysis

This analysis helps us identify the mutual interaction of the factors and their relative influence on the performance functions. It can also be used to decide the value of the factor for which the targeted performance can be achieved [10].

5.1 Analysis of Mean

The analysis of mean (ANOM) is carried out by calculating the mean of the performance values for each level of factors. These values are then plotted. The Figures in 3(a)-3(c) shows the level versus the plotted performance values for different values of reliability index (β), and the spread (S). In engineering applications the preferred values of reliability index are chosen as 3 or 3.8 which is 99.99% and 99.97% reliable [10]. Therefore we have chosen 3 and 3.8 as the reliability index for illustration. The spread values chosen are 10% and 20% of the nominal value of the factor. To keep the manuscript short, all curves are not included.

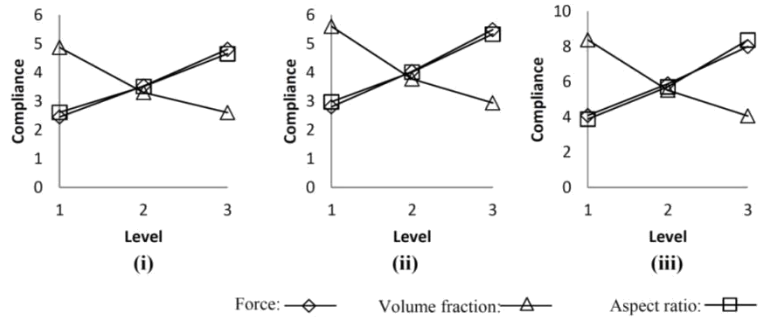


Fig. 3(a) For force inverter, ANOM of compliance considering, (i) deterministic, (ii) β=3, S=10%, (iii) β=3.8, S=20%

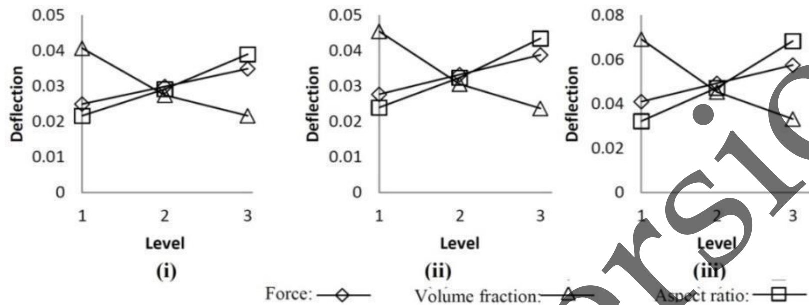


Fig. 3(b) For force inverter, ANOM of deflection (μm), considering, (i) deterministic, (ii) β=3, S=10%, (iii) β=3.8, S=20%

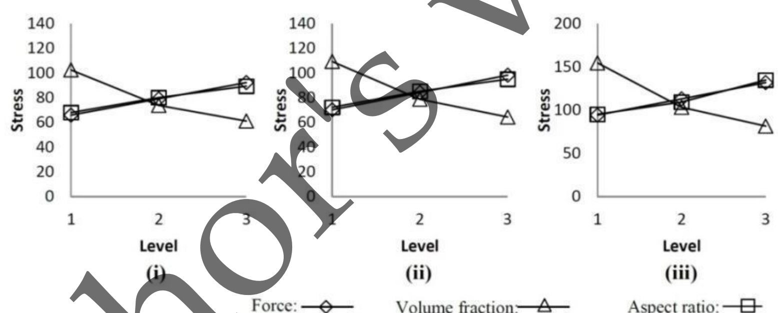


Fig. 3(c) For force inverter, ANOM of Von-Mises stress (N/m^2), (i) deterministic, (ii) β=3, S=10%, (iii) β=3.8, S=20%

Figures 3(a), 3(b) & 3(c) show the ANOM for the force inverter. It is observed that all three factors are dependent on one another. When performance function is compliance, volume fraction is the most significant factor and aspect ratio is the least significant factor. When performance function is deflection, volume fraction is the most significant factor and force is the least significant factor. Similarly, in the case of Von-Mises stress, volume fraction is the most influencing factor and force is the least influencing factor. However, when β=3.8 and S=20% alone, effect of aspect ratio is slightly higher than force for deflection. The reason for this change is the increased variation of the factor in the presence of uncertainties. For all performances, among all factors, volume fraction shows the most nonlinear effect, while force shows linear effect. It is also observed that with increasing value of reliability index and spread, the performance values also increases. This is because of the specific RBTO technique, discussed earlier.

In addition to these observations, it is also seen that with the increased values of reliability index and spread, the effects of aspect ratio and force come closer, when compliance is the performance function. While for deflection, the effect of aspect ratio and force became more distinct. This can be attributed to factor values that come from reliability index and spread. The observations made here can therefore be taken as guidelines to design a topologically optimized structure. It shows the different characteristics of factors under uncertainties.

5.2 Targeted Value Performances

From a designer's perspective, it is desired to achieve a targeted value of performance with high reliability.

However, when the selected RBTO scheme is applied to topology optimization problem, the value of performance functions are altered. In such a scenario, the intended performance cannot be achieved with desired reliability. To achieve a targeted performance with reliability, the design factor must be selected properly. This selection of controllable factors is done using mean performance value analysis. The mean values of performances are computed for each combination of factors. In Fig. 4, the mean compliance values with respect to the different combinations are shown. The mean compliance values are computed corresponding to the different values of reliability index and spread.

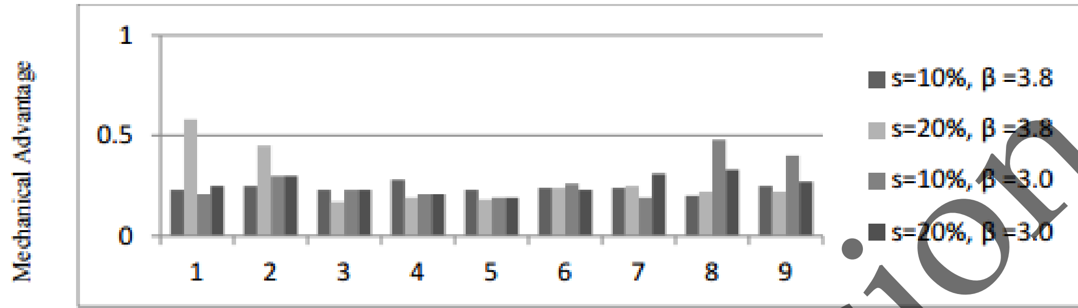


Fig. 4 Values of Mechanical Advantage for different values of S, β

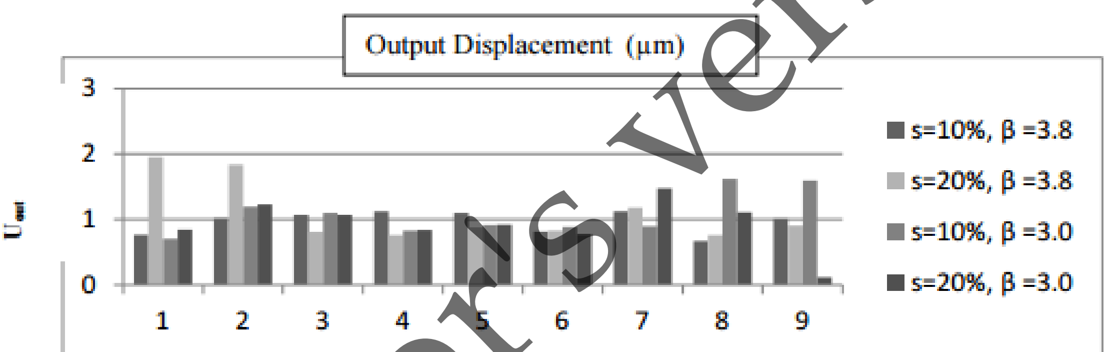


Fig. 5 Values of Output displacement for different values of S, β

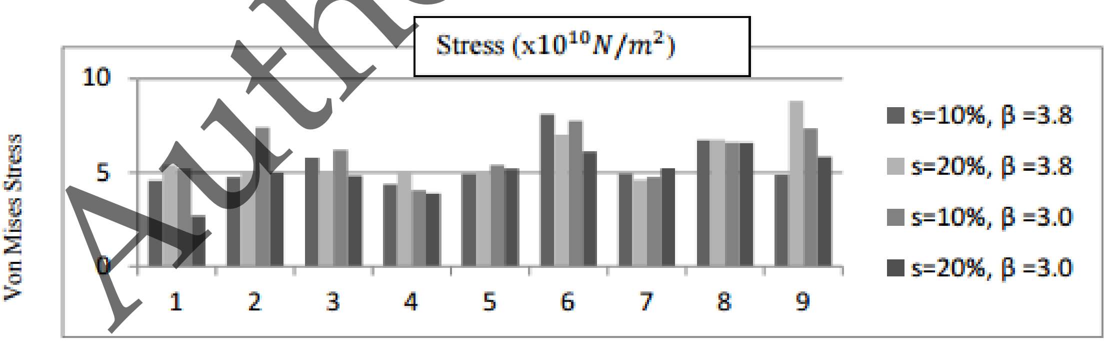


Fig. 6 Values of von Mises stress for different values of S, β

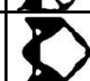
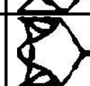
In Fig. 4, variation of compliance value with respect to different β and S can be seen corresponding to each combination. It is observed that with increased β and S values, the mean compliance increases. It can be observed that for a desired value of compliance or deflection, there are different combination of factors available. A targeted performance value can be achieved by selecting the available combination of controllable factors, reliability index, and spread value.

From Fig. 5, the variation of deflection value with respect to different β and S is seen corresponding to each combination. All observations are similar to that of compliance. It is observed that with increased β and S values, the mean deflection also increases. In order to observe the behavior of stress variations the mean values of maximum Von-Mises stress is presented in Fig. 6.

From Fig. 6, it is observed that with increasing values of β and S, the stress values also increases. The variation of stress over the different combinations is similar to that of compliance and deflection. Similar to the previous performance function, few observations are also made here. The changes in performance values are lesser with respect to β, compared to that with respect to S. In addition, the effect of change of S value is high when β =3.8 compared to that of at β =3. The observation regarding the performance verses β and S are because of their level values and the specific characteristic of RBTO method.

The optimal topologies along with the performance function values are shown in Table 2 for s=10%, β =3.8

Table 2. **Optimal topologies along with the performance function values for β=3.8**

S. No	*Force (N)*	*Volume Fraction*	*Number of elements in x- direction*	*Spread & Reliability*	*Mechanical Advantage*	*Output Displacement*	*Stress*	*Optimal Topologies*
1.	0.0010	0.30	30	10%, 3.8	0.23	0.7644	4.5789	
2.	0.0012	0.35	30	10%, 3.8	0.25	1.0153	4.7517	
3.	0.0014	0.40	30	10%, 3.8	0.23	1.0688	5.7813	
4.	0.0012	0.30	32	10%, 3.8	0.28	1.1246	4.3763	
5.	0.0014	0.35	32	10%, 3.8	0.23	1.1006	4.9356	
6.	0.0010	0.40	32	10%, 3.8	0.24	0.8127	8.100	
7.	0.0014	0.30	34	10%, 3.8	0.24	1.1279	4.9495	
8.	0.0010	0.35	34	10%, 3.8	0.20	0.6702	6.7190	
9.	0.0012	0.40	34	10%, 3.8	0.25	1.0126	4.8844	

6 Validation of Results with COMSOL

The maximum von Mises stress in the force inverter is found to be 2.3×10^4 when calculated using the MATLAB code. The optimal topology generated using MATLAB was imported to COMSOL

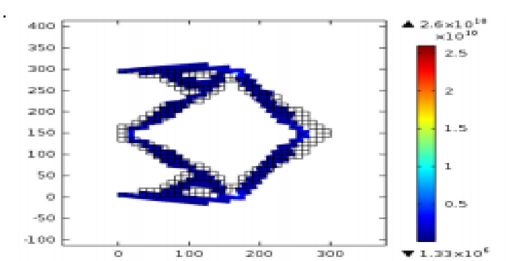


Fig. 7 von-Mises Stress diagram in COMSOL

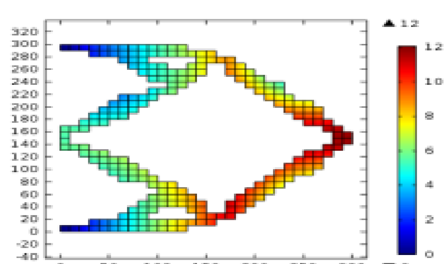


Fig. 8 Displacement diagram in COMSOL

For the same design factor values the force inverter was simulated in COMSOL. The results are shown in Fig. 7 and Fig. 8. The maximum stress as obtained in COMSOL is 2.6x10^4 *MPa*, which is close to the value computed using MATLAB.

7 Conclusions

The RBTO method assures the design of structural members against the realistic environment. However, to select the input parameter efficiently, the sensitivities of the factors are required to be known. In addition, the performance of the topology-optimized structure varies from its deterministic value, when RBTO is applied. In the present work, a methodology based on DOE and RBTO has been integrated to simulate the performance in realistic scenarios, in a desired domain of factors including the uncertainties of controllable and non-controllable factors. Current work also addresses the issues of achieving targeted performance for a given problem. The methodology has been illustrated using force inverter. The results are analyzed using ANOM. The sensitivity and the statistical significance of the factors are obtained. It is found that the volume fraction is the most significant factor. The equivalent equations are generated, which are useful to compute any compliance and deflection value within the selected range of factors. In addition, the presented methodology works well on coarse mesh, as long as it correctly represents the physics of the problem. It is because, the performance functions remains same, relative to the factorial design-combinations. Present analysis will be helpful to predict the behavior of performance function in realistic scenarios and to identify the relative robustness of the factor-combinations. Especially in the case of complex real time problems, where the input factor and uncertainties are difficult to directly relate with the output performances, this analysis can also be used to carry out targeted performance function problems of topology optimization.

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