Determination of workspace volume of parallel manipulators using Monte Carlo method

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Abstract. In this paper, we present a Monte Carlo simulation based method to determine the workspace of spatial parallel and hybrid manipulators. The method does not need the solution of the forward kinematics problem which is often difficult for spatial multi-degree-of-freedom parallel and hybrid manipulators. The method uses the solution of the inverse kinematics problem, which is often much simpler. The method can also readily incorporate joint limits and obtain the well-conditioned workspace. The approach is illustrated with a six-degree-of-freedom hybrid parallel manipulator which is a model for a human hand with three fingers. A typical human hand geometry and the range of motion at the joints are incorporated and the inverse kinematics equations for each finger is derived and used to obtain the volume of the hybrid parallel manipulator.

Key words: Workspace of parallel manipulators, Monte Carlo method, human hand inspired hybrid parallel manipulator

1 Introduction

The workspace of a parallel or a hybrid manipulator is much more difficult to find in comparison to that of a serial manipulator. In a serial manipulator, the workspace is determined by the geometry of the manipulator, its Denavit-Hartenberg parameters and the limits on the actuated joints. In a parallel or hybrid manipulators, in addition, the ranges of the motion of the passive joints need to be determined by solving the forward kinematics problem – if there are no real solutions to the forward kinematics problem, then the parallel manipulator cannot be assembled for the given actuated joint variables. Additionally, the self collisions of the links of the robot and the singularities which may split the workspace thereby restricting the motion across them increase the complexity of determining the workspace. Merlet [11, 12] summarizes the approaches for determining the workspace of parallel manipulators. These approaches are search based – an estimated region in space is discretized, the inverse kinematics is solved at discrete points to obtain the joint variable and then the joint variables are check for joint limit constraints. To obtain better resolution, the 3D workspace is discretized finer. One can also obtain the orientation workspace [7, 13] and also obtain regions where the manipulator Jacobian is not rank deficient [8]. In this work we use a Monte-Carlo based approach to obtain the well-conditioned workspace of a parallel hybrid manipulator. The main advantage of using Monte-Carlo based approach as described later, involves solving only the inverse kinematics problem for a manipulator and various other checks may be accommodated to ensure that the *well-conditioned* workspace is obtained without violating any joint limits. To illustrate the Monte-Carlo method based approach, we use a model of the human hand where the palm, the thumb, the index and the middle finger, grasping an object, is modeled as a hybrid parallel manipulator. There exists several models of multi-fingered human hand (see, for example, Stanford-JPL hand [15], Utah-MIT hand [9], DLR hand [1] and Metahand [2]). In this paper we present a six-degree-of-freedom model of a three-fingered hand, each finger with three degrees of freedom, with two joints actuated in each finger. For the kinematic model we use the anatomical dimensions of a typical human hand from available literature. The joint limit constraints in the fingers are also used in determining the workspace boundary and the volume. The Monte-Carlo based approach also uses the condition number of the Jacobian to determine the well-conditioned workspace. The paper is organized as follows. Section 2 gives a brief overview of the Monte-Carlo method and discusses why it may be useful for obtaining workspaces of manipulators. Section 3 describes the kinematic model of the hybrid parallel manipulator modeling the three-fingered human hand. In section 4 we describe two general results pertaining to the workspace of the manipulator and conclude with section 5 by summarizing the paper and proposing a possible avenue for future extension of the current work.

2 A review of the Monte Carlo method

The Monte-Carlo method can be used to evaluate integrals of arbitrary functions (vector or scalar, smooth or non-smooth) over an arbitrary domain [5]. The integral

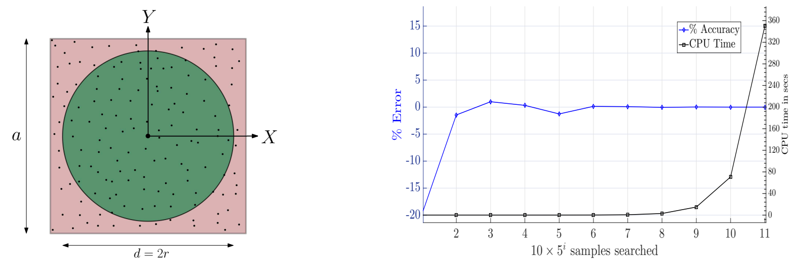
$$\mathscr{I} = \int_{[0,1]^d} f(\mathbf{x})\, \mathrm{d}\mathbf{x}$$

where $f(\cdot)$ is a bounded real valued function, can be obtained as $\mathbf{E}(f(\mathbf{U}))$ where $\mathbf{E}(\cdot)$ is the expectation of a variable taking a particular probabilistic value, and $\mathbf{U} = [u_1, u_2, ..., u_d]^T$ a $1 \times d$ vector taking random values of $u_i \in [0,1]\ \forall i = 1,2,...d$. From the strong law of large numbers the average,

$$S_N = \frac{1}{n}\sum_{i=1}^{n} f(U_i) \tag{1}$$

converges to $\mathbf{E}(f(\mathbf{U}))$ as $n \longrightarrow \infty$ with probability 1.0.

We use the Monte Carlo method to obtain the *well-conditioned* and *reachable* workspace of a parallel manipulator, by recognizing that it is an integration problem



(a) Visualization of Monte Carlo search

(b) Comparison of various Monte Carlo methods

Fig. 1: Demonstration of the Monte Carlo method

in $\Re^d$ where d is the dimensionality of the joint space of the parallel manipulator. For this we formulate a function $f(\mathbf{j}) = \begin{cases} 0 \\ 1 \end{cases}$, where $\mathbf{j} = \{\theta_i, \phi_j\}^T$, $\forall i = 1,2,..n$ actuated joint variables and $\forall j = 1,2,...m$ passive joint variables and $m+n = d$. At a given position and orientation of a chosen end-effector of the manipulator, the function f assumes either 0 or 1 depending on whether the said position and orientation of the parallel manipulator is well conditioned[1] and inverse kinematics of the manipulator is possible at that position and orientation with all the joint values within permissible joint limits.

We demonstrate the above by a following example. We assume that the well conditioned reachable workspace of a certain manipulator is a sphere with center at the origin $\{0,0,0\}$ and of radius r units. Therefore, the function $f(\mathbf{p}),\ p = \{x,y,z\}^T$ is used to classify whether a randomly selected point p is in, on or outside the permissible workspace. For this case, the check is very simple being, $f(\mathbf{p}) = \begin{cases} 1 & \forall\ x^2+y^2+z^2 \leq r^2 \\ 0 & otherwise \end{cases}$. We test the method by fixing $r = 2$ units and searching *uniformly* through a cube of sides $a = 5$ units, centered at the origin. A schematic view of the *workspace* and *search-space* is given in figure 1a. An approximation of the probability that a uniformly selected random point lies in or on the workspace is $\frac{N_{in}}{N_{total}}$ where N_{in} is the total number of points in/on the workspace (selected by ensuring $f(\mathbf{p}) = 1$), and N_{total} is the total number of points searched through. Since, by assumption, the points were randomly distributed, the volume of the workspace can be approximated by $V_W = \frac{N_{in}}{N_{total}} \times a^3$. A comparison of Monte Carlo methods with different iteration depths is given in figure 1b. We observe that the Monte Carlo method with $N_{total} = 10 \times 5^6 = 156,250$ samples is quite accu-

[1] We have used a definition of the condition number which encompasses both linear and angular motion of the manipulator at the said position and orientation. The *well conditioned*-ness is ensured by restricting the condition number to be less than 100 at all times.

rate (accuracy is $\geq 99.8\%$) and takes fairly low computation time[2] of less than 2 seconds.

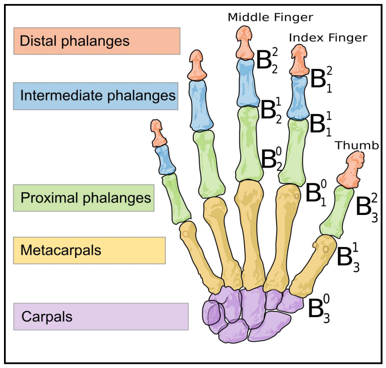
3 Description of the parallel manipulator

In this section, we first consider an anatomical representation of the human hand (see figure 2a) and then present a schematic representation of the proposed manipulator (see figure 2b). For the kinematic model, we consider only the thumb, index and middle finger. In figures 2a and 2b, all the joints of interest are labeled. For the index and middle fingers, the labels with a suffix 0, i.e. B_1^0 & B_2^0 represent the *metacarpo-phalangeal joints*, B_3^0 is the trapezium joint between the carpals and metacarpal bone of the thumb. For the index and middle fingers, the joints with suffix one, i.e. B_1^1 & B_2^1 are the joints between the proximal and intermediate phalanges, for the thumb, the joint B_3^1 indicates the joint between the metacarpal and the proximal phalanx bone. Finally, B_1^2 & B_2^2 indicate the joints between the intermediate and distal phalanges, for the thumb, the joint B_3^2 indicates the joint between proximal and distal phalanx of the thumb. The main difference between the proposed model and that of the Salisbury hand (see [15]) is that we are considering the *metacarpo-phalangeal* joint for the index and middle fingers to be a two degree of freedom joint, as opposed to a single revolute joint, as considered by Salisbury and others. The joint was realized by 2 intersecting orthogonal revolute joints. To obtain analytical solutions of the inverse kinematic problems of all the joint values during a given motion of the manipulator, we realize that we can have at most 9 joints with 6 active joints for the targeted 6 degrees of freedom and 3 passive joint, distributed as one passive joint per finger.

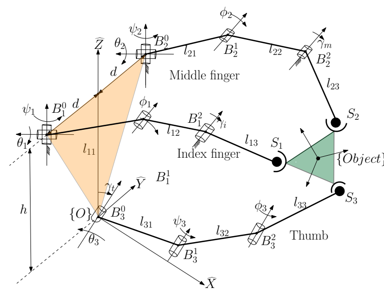
Kinesiological studies (see the work by Nakamura et al. [14] and the references contained therein for more details) have shown that all the joints in the human finger do not equally participate in the prehensile movements of the human hand. For a given grasping task, the motion is generally started from the proximal joints B_1^0 ,B_2^0 & B_3^0 and end in the distal joints B_1^2 ,B_2^2 & B_3^2, with the proximal joints being active for most of the time. Therefore, we choose the proximal joints to be actuated and we fix the distal joints of the index and middle fingers B_1^2 ,B_2^2 and make B_3^2 passive. We conservatively choose the joint limit ranges to be ranging from $0°$ to $90°$. This is somewhat less to that specified by Lin et al. [10], Degeorges and Oberlin [4], and Degeorges et al. [3]. This was done to exclude the joint values greater than $0°$ and less than $90°$, which may be introducing singularities, and increasing the computational time by checking the equivalent condition number for more number of points.

A brief formulation and solution of the inverse kinematics (IK) problem is given in appendix 1. It maybe noted that the inverse kinematics of the manipulator, for the

[2] The CPU times are for Matlab® R2015a running on a Windows 7 PC with an Intel XEON quad core processor at 3.10 GHz and 16 GB of RAM



(a) Anatomy of human hand {https://en.wikipedia.org/wiki/Hand}



(b) Schematic of the parallel manipulator

Fig. 2: Anatomical and schematic representation of the human hand

Table 1: Joint notations in figure 2b and permissible motions

Joint center	Joint variable	Nature	Range of motion/joint value
B_1^0 and B_2^0	θ_1 and θ_2	Active	$0°$ to $90°$
B_3^0	θ_3	Active	$-45°$ to $45°$
B_1^1 and B_2^1	ϕ_1 and ϕ_2	Active	$0°$ to $90°$
B_3^1	ψ_3	Active	$0°$ to $90°$
B_1^0 and B_2^0	ψ_1 and ψ_2	Passive	$0°$ to $15°$
B_3^2	ϕ_3	Passive	$0°$ to $90°$
B_1^2 and B_2^2	γ_i and γ_m	Fixed	$0°$
B_3^0	γ_t	Fixed	$\gamma_t = 45°$
S_1, S_2 and S_3	$\{\xi_X^i, \xi_Y^i\}\ \forall i = 1,2,3$	Passive	$\pm 45°$

index, middle and thumb, can be solved analytically since the eliminant obtained is a quartic function of the angle ψ_i (see, Ghosal [6]). The solution of the direct kinematics problem requires the solution of a sixteenth degree ploynomial.

4 Results: Workspaces of the manipulator

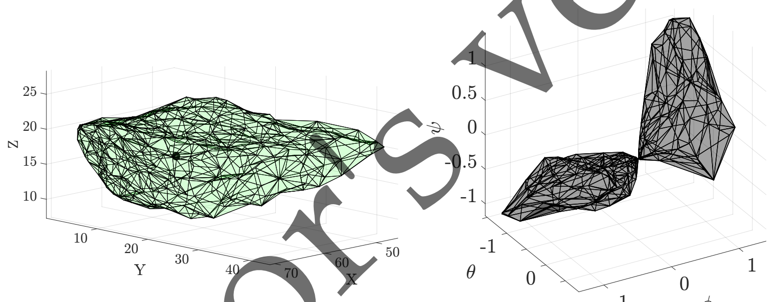
For simulation we use the following dimensions measured off the right hand of an adult male individual. The dimensions shown in table 2, along with the abbreviations used correspond to the same in figure 2b. For determining the workspace of the manipulator, we have considered 200,000 random points in the Cartesian space bounded by $X \in [0,80]$mm, $Y \in [0,80]$mm and $Z \in [0,80]$mm. At each of these points we have assigned a random configuration of the object, $\triangle S_1S_2S_3$ in figure 2b and checked the inverse kinematics solution of the manipulator. If the IK problem was solvable by satisfying the joint limits in table 1, the *equivalent condition num-*

ber[3] was less than 10^2 and the motions of the S joints were within the prescribed limits, the point is counted and used for the representation as shown in figure 3a. Using the data from table 2 we obtain the volume of the workspace of the manip-

Table 2: Sample finger and hand segment lengths

Hand part	**Symbols in figure 2b**	**Values in mm.**
Index finger	$\{l_{11}, l_{12}, l_{13}\}$	{35.45, 23.92, 17.6}
Middle finger	$\{l_{21}, l_{22}, l_{23}\}$	{41.33, 22.3, 18.26}
Thumb	$\{l_{31}, l_{32}, l_{33}\}$	{45.7, 36.23, 20.52}
Palm	$\{d, h\}$	{15, 68.83}

ulator as 1.4×10^3 mm^3. The orientations workspace, in terms of $X - Y - Z$ Euler angles, at a point (marked by a black dot) is shown in figure 3b. The shape and volume of the workspaces shown in figure 3 was obtained in less than 50 seconds. It may be noted that the range of the Euler angles are chosen to be $\pm 90°$. The top part of figure 4 shows the workspace of the Salisbury hand ([15]) for the same set of parameters and it can be seen that the *well conditioned* workspace for the proposed manipulator is larger than the workspace of the Salisbury hand.



(a) The positional workspace

(b) The orientation workspace (axes in radians)

Fig. 3: Position and orientation workspaces of the manipulator

5 Conclusions

In this paper, we have used the Monte-Carlo method to determine the workspace of a six-degree-of-freedom hybrid-parallel manipulator. The hybrid-parallel manipula-

[3] Obtained by combining the linear and angular velocity Jacobian matrices by scaling the lengths by $\{l_{11} + l_{12} + l_{13}\}$ as shown in figure 2b.

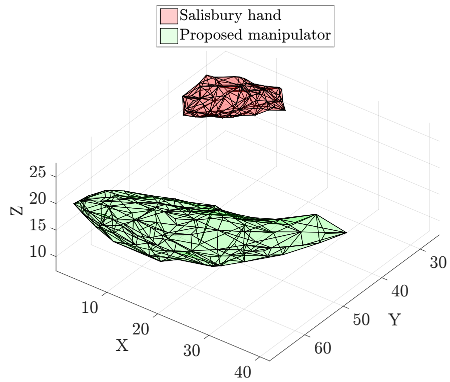


Fig. 4: Comparison of well-conditioned workspaces between the proposed manipulator and the Salisbury hand (see [15])

tor is a model of a three-fingered human hand grasping an object with the contact between the object and the fingers modeled with spherical joint which implies that there is no rolling at the contact. Each finger has two actuated and one passive joint. The dimensions of the link, the geometry and the joint limits of the hybrid-parallel manipulator are derived from a typical human hand. The general shape and measure of the workspace has been obtained using the Monte Carlo method. However, a majority of dexterous manipulation tasks are realized by rolling type of contact between the finger-tips and the object, and we are attempting to extend this approach to include rolling contact between the fingers and the object.

1 Appendix I: Solution of the IK problem of the proposed manipulator

For the most general case, the position vector of the point S_1 (see figure 2a) is given as the expressions of X, Y and Z below. From the expressions in equations 2, 3 and 4 we obtain $E_1 = X^2 + (Y+d)^2 + (Z-h)^2$ as given in equation 5. Using the expressions for E_1 and Z from equations 5 and 4, in Sylvester's dialytic method we can obtain the eliminant for ψ_1 as a quartic function of the angular variable. The value of θ_1 may be obtained by solving the expression for $-X + (Y+d)$ symbolically and the value of ϕ_1 is obtained by using terms from the expressions of Z and E_1 as discussed in [6].

$$X = \frac{1}{2}\left(l_{11}\cos(\psi_1 - \theta_1) + l_{11}\cos(\psi_1 + \theta_1) + l_{12}\cos(\phi_1 - \psi_1 + \theta_1) + l_{12}\cos(\phi_1 + \psi_1 + \theta_1) + l_{13}\cos(\gamma_i + \phi_1 - \psi_1 + \theta_1) + l_{13}\cos(\gamma_i + \phi_1 + \psi_1 + \theta_1)\right) \quad (2)$$

$$Y = \frac{1}{2}\left(l_{11}\sin(\psi_1+\theta_1) + l_{11}\sin(\psi_1-\theta_1) + l_{12}\sin(\phi_1+\psi_1+\theta_1) - l_{12}\sin(\phi_1-\psi_1+\theta_1) + l_{13}\sin(\gamma_i+\phi_1+\psi_1+\theta_1) - l_{13}\sin(\gamma_i+\phi_1-\psi_1+\theta_1)\right) - d \tag{3}$$

$$Z = -\sin(\phi_1+\gamma_i+\theta_1)\,l_{13} - \sin(\theta_1+\phi_1)\,l_{12} - \sin(\theta_1)\,l_{11} + h \tag{4}$$

$$E_1 = (2\cos(\gamma_i)\,l_{11}l_{13} + 2\,l_{12}l_{11})\cos(\phi_1) - 2\,l_{13}\sin(\gamma_i)\sin(\phi_1)\,l_{11} + 2\,l_{13}\cos(\gamma_i)\,l_{12} + {l_{11}}^2 + {l_{12}}^2 + {l_{13}}^2 \tag{5}$$

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