Kinematic and dynamic modeling and base inertial parameters determination of the Quadrupteron parallel manipulator

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Abstract. This paper deals with the kinematic analysis, dynamic modeling and base inertial parameter determination of a member of multipteron parallel manipulator family, namely, Quadripteron. First, as a prerequisite for dynamic analysis, kinematic relations are obtained. By using a new geometric approach, the solution of the inverse kinematic problem is made equivalent to solve the problem of determining the intersection of two circles within a plane. Compared to other proposed methods, this approach yields more compact and closed-form solutions. The instantaneous kinematic problem is solved via employing the screw theory. Based on foregoing kinematic relations and the concept of link Jacobian matrices, the dynamic model is formulated by means of the principle of virtual work. Furthermore, in order to obtain a more compact formulation for the dynamic analysis, a reduced dynamic model is obtained by determining the base inertial parameters of the under study manipulators.

Key words: Parallel robots, Kinematics, Screw theory, Dynamic model, Base inertial parameters

1 Introduction

It is well known that, compared to serial robots, parallel manipulators can offer several advantages in terms of better rigidity, higher precision and better dynamic performances. Due to the widespread application of industrial robots performing Schönflies motion pattern, several researches have been conducted on the synthesis and prototyping of parallel or hybrid manipulators featuring the Schönflies motions.

The H4 robot, a fully parallel Schönflies motion generator, was introduced [1]. Also, the (fully parallel) Kanuk and the (hybrid) Manta architectures were proposed [2]. All of the aforementioned architectures were developed mainly based on intuition. In [3], a synthesis method based on screw theory was presented and a large number of other new architectures were discovered. In [4], a quasi-decoupled 4-DOF Schönflies motion generator was proposed, based on the type synthesis presented in [3]. This architecture, referred to as the Quadrupteron, is of the 4-$\underline{\text{P}}$RRU type. Here and throughout this paper, in order to represent the kinematic arrangement of a limb, P, R and U stand for a revolute, prismatic and universal joints, respectively, where the actuated one is underlined.

Several studies concerning the Quadrupteron have been carried out over the last decade, however, most of researches are related to their kinematic properties, namely, direct and inverse kinematics, workspace and singularity analysis [4–6]. While the kinematic analysis is an essential and indispensable step in studying a multibody system, in many applications such as simulation and model-based control strategies, an accurate knowledge of the dynamic behavior of the manipulator is a definite asset. To the best knowledge of the authors, as far as Quadrupteron is concerned, there is still a gap on the dynamic analysis of this type of mechanism. There are several approaches for formulating the dynamic model of a multibody system, some of which are: Newton-Euler, the Euler-Lagrange formulation, the principle of virtual work, Kane's method and Natural Orthogonal Complement (NOC) approach [7].

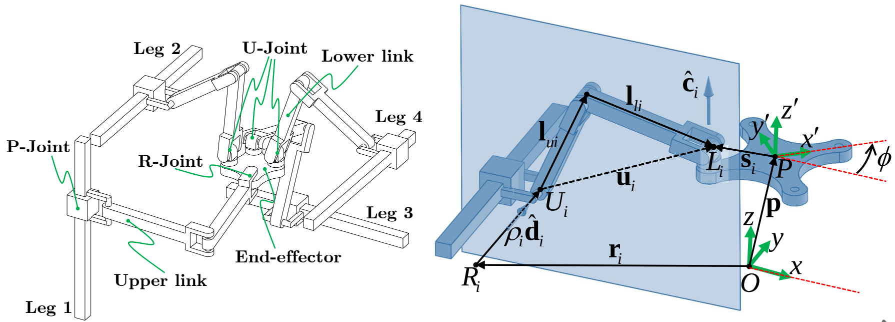
While the mathematical structure of the dynamic model can be formulated with the above-mentioned approaches, one of the main factors affecting the accuracy of the results, is the exactness of the values of the physical parameters used in the model. It is well-known that not all the inertial parameters have a direct effect on the dynamic response of the system. Therefore, only a set of identifiable parameters can be estimated. The minimal set of identifiable parameters, which are often referred to as *base inertial parameters*, can be determined symbolically or numerically [8, 9]. The determination of the base inertial parameters also contributes in reducing the computational cost of the dynamic models, as it eliminates or groups the original inertial parameters [8].

The main contribution of the this paper can be regarded as: 1) Proposing a new geometric approach to solve the position analysis of the under study manipulators which leads to a compact solution for the inverse position problem. 2) Obtaining the dynamic model of a member of multipteron parallel manipulator family, namely, Quadrupteron, in a closed and unified form. 3) Minimizing the computational cost of the dynamic model by obtaining the base inertial parameters of the under study manipulator and reducing the dynamic models without loosing the accuracy of the models.

2 Position analysis

The Quadrupteron, represented schematically in Fig. 1(a), is a 4-DoF parallel mechanism capable of producing the Schönflies motions. The Quadrupteron is composed of 4 legs of the PRRU type attached to an end-effector. In one of the legs (Leg 1 in Fig. 1)(a), the last U joint degenerates into an R joint.

In this section, as the first step of obtaining the kinematic relationships, the Inverse Displacement Problem (IDP) is addressed. Even though the Quadrepteron have been studied before [5, 6], a simple closed-form analytical solution is clearly preferred. Such a solution is not only more efficient with regard to computational cost, but also gives a valuable geometric insight for the design. In this regard, a new geometrical method is proposed which results in a general closed-form solutions for



(a) An overall schematic of the manipulator (b) Kinematic model of an arbitrary leg

Fig. 1: The Quadrupteron: a 4-DOF Schönflies-motion parallel mechanism.

the IDP. Before proceeding with the analysis, the following lemma is presented:

Lemma 1. *Suppose* $\hat{\mathbf{d}}$ *is a known unit vector and* $\mathbf{u}$, $\mathbf{l}_u$ *and* $\mathbf{l}_l$ *are vectors in a plane perpendicular to* $\hat{\mathbf{d}}$, *satisfying* $\mathbf{l}_u + \mathbf{l}_l = \mathbf{u}$. *Assuming that* $\mathbf{u}$ *and length of* $\mathbf{l}_u$ *and* $\mathbf{l}_l$ *are known, there are two possible solutions for* $\mathbf{l}_u$ *and* $\mathbf{l}_l$*:*

$$\mathbf{l}_u = \frac{1}{2\|\mathbf{u}\|}\Big\{\left(\|\mathbf{u}\|^2 + {l_u}^2 - {l_l}^2\right)\hat{\mathbf{u}} \pm \\ \sqrt{(\|\mathbf{u}\| + l_u + l_l)(-\|\mathbf{u}\| + l_u + l_l)(\|\mathbf{u}\| - l_u + l_l)(\|\mathbf{u}\| + l_u - l_l)}\,(\hat{\mathbf{d}} \times \hat{\mathbf{u}})\Big\} \tag{1}$$

$$\mathbf{l}_l = \frac{1}{2\|\mathbf{u}\|}\Big\{\left(\|\mathbf{u}\|^2 + {l_l}^2 - {l_u}^2\right)\hat{\mathbf{u}} \mp \\ \sqrt{(\|\mathbf{u}\| + l_u + l_l)(-\|\mathbf{u}\| + l_u + l_l)(\|\mathbf{u}\| - l_u + l_l)(\|\mathbf{u}\| + l_u - l_l)}\,(\hat{\mathbf{d}} \times \hat{\mathbf{u}})\Big\} \tag{2}$$

where l_u *and* l_l *are respectively the length of* $\mathbf{l}_u$ *and* $\mathbf{l}_l$[1].

Remark 1: The solution given in Lemma 1 is the same as finding the intersection of two circles in given plane with known diameters.
Remark 2: If the expression under the radical sign in Eqs. (1) and (2) become negative there is no real solution for $\mathbf{l}_u$ and $\mathbf{l}_l$. From a geometrical standpoint, this condition takes place when the two circles have no intersection.

In what follows, we will illustrate how Lemma 1 is used to solve the IDP of the under study manipulators. Referring to Fig. 1(b), the following equation can be established for the i^{th} leg:

$$\mathbf{r}_i + \rho_i\hat{\mathbf{d}}_i + \mathbf{u}_i = \mathbf{p} + \mathbf{s}_i \tag{3}$$

[1] This lemma can be easily verified by substituting Eqs. (1) and (2) into $\mathbf{l}_u + \mathbf{l}_l = \mathbf{u}$, $\hat{\mathbf{d}}.\mathbf{l}_u = \hat{\mathbf{d}}.\mathbf{l}_l = \hat{\mathbf{d}}.\mathbf{u} = 0$ and $\frac{l_u}{\|\mathbf{l}_u\|} = \frac{l_l}{\|\mathbf{l}_l\|} = 1$. To the best knowledge of the authors, content of this lemma is not available in the literature.

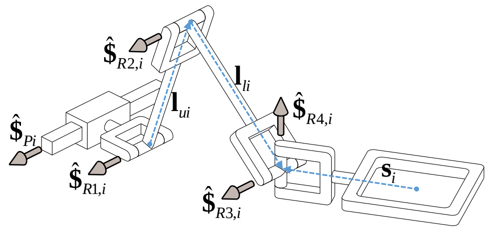


Fig. 2: Screw axes associated with P̲RR(RR) kinematic structure.

By dot multiplying both sides of Eq. (3) with $\hat{\mathbf{d}}_i$ and considering the fact that $\mathbf{u}_i$ is perpendicular to $\hat{\mathbf{d}}_i$, the following equation is obtained:

$$\rho_i = (\mathbf{p} + \mathbf{s}_i - \mathbf{r}_i) . \hat{\mathbf{d}}_i \tag{4}$$

The latter equation represents the relationship between the pose of the end-effector and position of the i^{th} actuated P-joint. In addition, by substituting Eq. (4) into Eq. (3), $\mathbf{u}_i$ can be obtained as:

$$\mathbf{u}_i = \left(\mathbb{1}_{3\times 3} - \hat{\mathbf{d}}_i \hat{\mathbf{d}}_i^T\right)(\mathbf{p} + \mathbf{s}_i - \mathbf{r}_i) \tag{5}$$

According to Fig. 1(b), $\mathbf{u}_i$, $\mathbf{l}_{ui}$ and $\mathbf{l}_{li}$ are vectors in a plane perpendicular to $\hat{\mathbf{d}}_i$, satisfying $\mathbf{l}_u + \mathbf{l}_l = \mathbf{u}$. Hence, by using Lemma 1 one can obtain $\mathbf{l}_{ui}$ and $\mathbf{l}_{li}$ as:

$$\mathbf{l}_{ui} = \frac{1}{2\|\mathbf{u}_i\|}\Big\{ \left(\|\mathbf{u}_i\|^2 + {l_{ui}}^2 - {l_{li}}^2\right)\hat{\mathbf{u}}_i \pm \\ \sqrt{(\|\mathbf{u}_i\| + l_{ui} + l_{li})(-\|\mathbf{u}_i\| + l_{ui} + l_{li})(\|\mathbf{u}_i\| - l_{ui} + l_{li})(\|\mathbf{u}_i\| + l_{ui} - l_{li})}\left(\hat{\mathbf{d}}_i \times \hat{\mathbf{u}}_i\right)\Big\} \tag{6}$$

$$\mathbf{l}_{li} = \frac{1}{2\|\mathbf{u}_i\|}\Big\{ \left(\|\mathbf{u}_i\|^2 + {l_{li}}^2 - {l_{ui}}^2\right)\hat{\mathbf{u}}_i \mp \\ \sqrt{(\|\mathbf{u}_i\| + l_{ui} + l_{li})(-\|\mathbf{u}_i\| + l_{ui} + l_{li})(\|\mathbf{u}_i\| - l_{ui} + l_{li})(\|\mathbf{u}_i\| + l_{ui} - l_{li})}\left(\hat{\mathbf{d}}_i \times \hat{\mathbf{u}}_i\right)\Big\} \tag{7}$$

3 Instantaneous kinematics analysis

One of the requirements for obtaining the dynamic model by using the virtual work principal, is to derive the relationship between the twist of all of the manipulator's parts with a suitable reference, such as twist of the end-effector. In this section, by employing the screw theory, the instantaneous twist of each link and the input velocities will be calculated with respect to end-effector's twist.

According to Fig. 2, which depicts a kinematic chain with P̲RR(RR) structure which resembles the i^{th} leg of Quadrupteron, the unit joints screws of the i^{th} leg can be written as:

$$\begin{aligned} \hat{\$}_{Pi} &= \begin{bmatrix} \mathbf{0} \\ \hat{\mathbf{d}}_i \end{bmatrix}; \quad \hat{\$}_{R1,i} = \begin{bmatrix} \hat{\mathbf{d}}_i \\ (\mathbf{s}_i - \mathbf{l}_{ui} - \mathbf{l}_{li}) \times \hat{\mathbf{d}}_i \end{bmatrix}; \hat{\$}_{R2,i} = \begin{bmatrix} \hat{\mathbf{d}}_i \\ (\mathbf{s}_i - \mathbf{l}_{li}) \times \hat{\mathbf{d}}_i \end{bmatrix} \\ \hat{\$}_{R3,i} &= \begin{bmatrix} \hat{\mathbf{d}}_i \\ \mathbf{s}_i \times \hat{\mathbf{d}}_i \end{bmatrix}; \qquad \hat{\$}_{R4,i} = \begin{bmatrix} \hat{\mathbf{c}}_i \\ \mathbf{s}_i \times \hat{\mathbf{c}}_i \end{bmatrix}; \end{aligned} \tag{8}$$

Now, considering each branch as an open-loop chain and expressing the instantaneous twist of the end-effector, $\$_E$, in terms of the joint screws, gives:

$$\$_E = \hat{\$}_{Pi}\dot{\rho}_i + \hat{\$}_{R1,i}\dot{\theta}_{R1,i} + \hat{\$}_{R2,i}\dot{\theta}_{R2,i} + \hat{\$}_{R3,i}\dot{\theta}_{R3,i} + \hat{\$}_{R4,i}\dot{\theta}_{R4,i} \tag{9}$$

In order to obtain the relationship between the output twist, $\$_E$, and the input joint velocities, one should eliminate the passive joint screws from Eq. (9). To do so, both sides of Eq. (9) is left multiplied by a wrench, reciprocal to the passive joints, i.e., $\boldsymbol{\xi}_i^{\mathrm{T}} = \begin{bmatrix} (\mathbf{s}_i \times \hat{\mathbf{d}}_i)^{\mathrm{T}} & \hat{\mathbf{d}}_i^{\mathrm{T}} \end{bmatrix}$. Hence, the relationship between the twist of the end-effector and the linear velocity of the prismatic joints can be obtained as:

$$\begin{bmatrix} \dot{\rho}_1 \\ \vdots \\ \dot{\rho}_4 \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{d}}_1^{\mathrm{T}} & (\hat{\mathbf{d}}_1 \times \hat{\mathbf{c}}_1).\mathbf{s}_1 \\ \vdots & \vdots \\ \hat{\mathbf{d}}_4^{\mathrm{T}} & (\hat{\mathbf{d}}_4 \times \hat{\mathbf{c}}_4).\mathbf{s}_4 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\phi} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\phi} \end{bmatrix} \tag{10}$$

where $\mathbf{J}$ is called the *input-output Jacobian matrix*.
Also, taking the time derivative of Eq. (3) and dot multiplying both sides of the resulting equation by $\mathbf{l}_{li}$ and $\mathbf{l}_{ui}$ results in:

$$\begin{bmatrix} \mathbf{v}_{Ui} \\ \dot{\theta}_{ui}\hat{\mathbf{d}}_i \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{d}}_i\hat{\mathbf{d}}_i^{\mathrm{T}} & \hat{\mathbf{d}}_i\hat{\mathbf{d}}_i^{\mathrm{T}}(\hat{\mathbf{c}}_i \times \mathbf{s}_i) \\ \dfrac{\hat{\mathbf{d}}_i\mathbf{l}_{li}{}^{\mathrm{T}}}{(\mathbf{l}_{ui} \times \mathbf{l}_{li}).\hat{\mathbf{d}}_i} & \dfrac{\hat{\mathbf{d}}_i\mathbf{s}_i{}^{\mathrm{T}}(\mathbf{l}_{li} \times \hat{\mathbf{c}}_i)}{(\mathbf{l}_{ui} \times \mathbf{l}_{li}).\hat{\mathbf{d}}_i} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\phi} \end{bmatrix} = \mathbf{J}_{ui} \begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\phi} \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} \mathbf{v}_{Li} \\ \dot{\theta}_{li}\hat{\mathbf{d}}_i \end{bmatrix} = \begin{bmatrix} \mathbb{1}_{3\times 3} & \hat{\mathbf{k}} \times \mathbf{s}_i \\ \dfrac{\hat{\mathbf{d}}_i\mathbf{l}_{ui}{}^{\mathrm{T}}}{(\mathbf{l}_{li} \times \mathbf{l}_{ui}).\hat{\mathbf{d}}_i} & \dfrac{\hat{\mathbf{d}}_i\mathbf{s}_i{}^{\mathrm{T}}(\mathbf{l}_{ui} \times \hat{\mathbf{c}}_i)}{(\mathbf{l}_{li} \times \mathbf{l}_{ui}).\hat{\mathbf{d}}_i} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\phi} \end{bmatrix} = \mathbf{J}_{li} \begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\phi} \end{bmatrix} \tag{12}$$

where $\mathbf{J}_{ui}$ and $\mathbf{J}_{li}$ are respectively the i^{th} *upper and lower link Jacobian matrices*. Also, $\dot{\theta}_{ui}$ and $\dot{\theta}_{ui}$ are respectively the magnitude of angular velocities of the i^{th} upper and lower link.

4 Dynamics analysis

In this section, the dynamic model of the under study manipulators are formulated by means of d'Alembert's form of the principle of virtual work. Figure 3 depicts the coordinate frames attached to the i^{th} upper and lower links. The position of the center

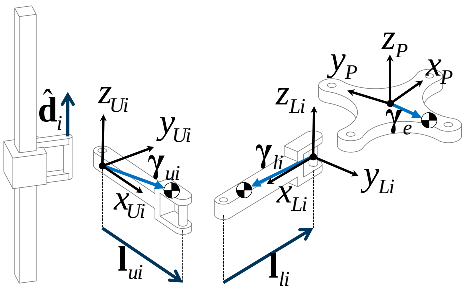


Fig. 3: Local coordinate frames assigned to each link.

of mass of the i^{th} upper link, lower links and end-effector relative to their reference points are respectively denoted by $\boldsymbol{\gamma}_{ui}$, $\boldsymbol{\gamma}_{li}$ and $\boldsymbol{\gamma}_e$. Assuming that the friction forces and torques at the joints are negligible, using the principle of virtual work,referring to the manipulator Jacobian matrix, given in Eq. (10), and link Jacobian matrices given in Eqs. (11) and (12), dynamics of the under study manipulator can be stated as:

$$\mathcal{F}_a = -\tilde{\mathcal{F}}_p - \mathbf{J}^{-\mathrm{T}}\Big(\sum_{i=1}^{4} \mathbf{J}_{ui}{}^{\mathrm{T}}\mathcal{F}_{ui} + \sum_{i=1}^{4} \mathbf{J}_{li}{}^{\mathrm{T}}\mathcal{F}_{li} + \mathcal{F}_e\Big) \tag{13}$$

where $\mathcal{F}_a = \begin{bmatrix} F_{a1} & \cdots & F_{aj} \end{bmatrix}^{\mathrm{T}}$ is the vector of input forces and $\tilde{\mathcal{F}}_p$ is:

$$\tilde{\mathcal{F}}_p = \begin{bmatrix} m_{p1}(\hat{\mathbf{d}}_1.\mathbf{g} - \ddot{\rho}_1) & \ldots & m_{pj}(\hat{\mathbf{d}}_4.\mathbf{g} - \ddot{\rho}_4) \end{bmatrix}^{\mathrm{T}} \tag{14}$$

And $\mathcal{F}_{ui}$, $\mathcal{F}_{li}$ and $\mathcal{F}_e$ represent the resultant of applied and inertia forces exerted to the reference point of the i^{th} upper link, i^{th} lower link and the end-effector.

Equation (13) denotes the relation between the actuators' forces and the applied and inertia wrenches acting on the manipulator.

Now by using a method based on principle of virtual work [10], the dynamic model given in Eq. (13) is rewritten in a linear form:

$$\mathcal{F}_a = \mathbf{J}^{-\mathrm{T}}\begin{bmatrix} \mathbf{J}^{\mathrm{T}}\boldsymbol{\Omega}_p & \mathbf{J}_{u1}{}^{\mathrm{T}}\boldsymbol{\Omega}_{u1} & \cdots & \mathbf{J}_{uj}{}^{\mathrm{T}}\boldsymbol{\Omega}_{uj} & \mathbf{J}_{l1}{}^{\mathrm{T}}\boldsymbol{\Omega}_{l1} & \cdots & \mathbf{J}_{lj}{}^{\mathrm{T}}\boldsymbol{\Omega}_{lj} & \boldsymbol{\Omega}_e \end{bmatrix}\mathbf{P} \tag{15}$$

where $\boldsymbol{\Omega}_p$, $\boldsymbol{\Omega}_{ui}$, $\boldsymbol{\Omega}_{li}$ and $\boldsymbol{\Omega}_e$ are matrices which are functions of kinematic properties of the manipulator and $\mathbf{P} = \begin{bmatrix} \mathbf{p}_p & \mathbf{p}_{u1} & \ldots & \mathbf{p}_{uj} & \mathbf{p}_{l1} & \ldots & \mathbf{p}_{lj} & \mathbf{p}_e \end{bmatrix}$ in which the entries are defined as:

$$\mathbf{p}_p = \begin{bmatrix} m_{p1} & \cdots & m_{pj} \end{bmatrix}^{\mathrm{T}};\ \mathbf{p}_{ui} = \begin{bmatrix} m_{ui} \\ m_{ui}{}^{Ui}\boldsymbol{\gamma}_{ui} \\ {}^{Ui}I_{ui(z)} \end{bmatrix};\ \mathbf{p}_{li} = \begin{bmatrix} m_{li} \\ m_{li}{}^{Li}\boldsymbol{\gamma}_{li} \\ {}^{Li}I_{li(z)} \end{bmatrix};\ \mathbf{p}_e = \begin{bmatrix} m_e \\ m_e{}^{P}\boldsymbol{\gamma}_e \\ {}^{P}I_{e(z)} \end{bmatrix}; \tag{16}$$

Table 1: Base inertial parameters of the Quadrupteron parallel manipulator.

Base Inertial Parameters	Linear Combination	Base Inertial Parameters	Linear Combination
P_{b1}	$m_{p1}+m_{u1}+20.11^{L1}I_{l1(z)}$	P_{b14}	${}^{U4}I_{u4(z)}+0.91^{L4}I_{l4(z)}$
P_{b2}	$m_{p2}+m_{u2}+38.58^{L2}I_{l2(z)}$	P_{b15}	$m_{l1}-20.11^{L1}I_{l1(z)}+m_{l4}-38.58^{L4}I_{l4(z)}+12.5m_e{}^{P}\gamma_{e(x)}-12.5m_e{}^{P}\gamma_{e(y)}+312.5^{P}I_{e(z)}$
P_{b3}	$m_{p3}+m_{u3}+35.86^{L3}I_{l3(z)}$	P_{b16}	$m_{l1}{}^{L1}\gamma_{l1(x)}+4.48^{L1}I_{l1(z)}$
P_{b4}	$m_{p4}+m_{u4}+38.58^{L4}I_{l4(z)}$	P_{b17}	$m_{l1}{}^{L1}\gamma_{l1(y)}$
P_{b5}	${}^{U1}I_{u1(z)}+0.83^{L1}I_{l1(z)}$	P_{b18}	$m_{l2}-38.58^{L2}I_{l2(z)}-m_{l4}+38.58^{L4}I_{l4(z)}-25m_e{}^{P}\gamma_{e(x)}$
P_{b6}	$m_{u2}{}^{U2}\gamma_{u2(x)}+5.94^{L2}I_{l2(z)}$	P_{b19}	$m_{l2}{}^{L2}\gamma_{l2(x)}+6.21^{L2}I_{l2(z)}$
P_{b7}	$m_{u2}{}^{U2}\gamma_{u2(y)}$	P_{b20}	$m_{l2}{}^{L2}\gamma_{l2(y)}$
P_{b8}	${}^{U2}I_{u2(z)}+0.91^{L2}I_{l2(z)}$	P_{b21}	$m_{l3}-35.85^{L3}I_{l3(z)}+m_{l4}-38.58^{L4}I_{l4(z)}+12.5m_e{}^{P}\gamma_{e(x)}+12.5m_e{}^{P}\gamma_{e(y)}+312.5^{P}I_{e(z)}$
P_{b9}	$m_{u3}{}^{U3}\gamma_{u3(x)}+5.99^{L3}I_{l3(z)}$	P_{b22}	$m_{l3}{}^{L3}\gamma_{l3(x)}+5.99^{L3}I_{l3(z)}$
P_{b10}	$m_{u3}{}^{U3}\gamma_{u3(y)}$	P_{b23}	$m_{l3}{}^{L3}\gamma_{l3(y)}$
P_{b11}	${}^{U3}I_{u3(z)}+{}^{L3}I_{l3(z)}$	P_{b24}	$m_{l4}{}^{L4}\gamma_{l4(x)}+6.21^{L4}I_{l4(z)}$
P_{b12}	$m_{u4}{}^{U4}\gamma_{u4(x)}+5.94^{L4}I_{l4(z)}$	P_{b25}	$m_{l4}{}^{L4}\gamma_{l4(y)}$
P_{b13}	$m_{u4}{}^{U4}\gamma_{u4(y)}$	P_{b26}	$m_e-625^{P}I_{e(z)}$

5 Base inertial parameter determination

The dynamic model given in Eq. (15) is linear with respect to inertial parameters and it can be rewritten as $\boldsymbol{\tau}=\boldsymbol{\mathcal{D}}\mathbf{P}$, where $\mathbf{P}$ is the vector of inertia parameters and $\boldsymbol{\mathcal{D}}$ is called the dynamic matrix. As aforementioned, not all of the parameters will directly affect the dynamic model. Thus, by eliminating or grouping the parameters, one can reduce the number of inertial parameters. This reduced set of parameters is known as the *base inertial parameters*. In this section, the SVD-based approach given in [8] was used to determine the base inertial parameters of the Quadrupteron manipulator. The relation between the base inertial parameters and the original parameters is shown in Tables 1. It should be noted that, the parameters given in the aforementioned tables are not the only possible set for base inertial parameters and any invertible linear combination of them can be regarded as a new set of base inertial parameters.

By using the base inertial parameters, the dynamic model represented by Eq. (15) is reduced to $\boldsymbol{\tau}=\boldsymbol{\mathcal{D}}_{\text{red}}\mathbf{P}_{\text{red}}$, where $\boldsymbol{\mathcal{D}}_{\text{red}}$ is the reduced dynamic matrix after eliminating and grouping the inertial parameters and $\mathbf{P}_{\text{red}}$ is the vector containing the base inertial parameters. It is worth mentioning that by comparing the computational time of the reduced dynamic model with the complete dynamic model, it follows that the reduced dynamic model is approximately 41% faster than the original virtual work model.

6 Conclusion

In this paper, the kinematic and dynamic model of Quadrupteron parallel manipulator was derived. As a prerequisite to dynamic analysis, the kinematic analysis was performed which was investigated by resorting to the screw theory. The reason for which screw theory was adopted as kinematic investigation tool is that it provides a Jacobian-base formulation for mapping of the time rate changes of all joints, including passive and actuated, which is essential for dynamic analysis based on virtual work concept. Also, a new geometrical approach based on the intersection of two circles within a plane, was presented which resulted in a compact closed-form solution for inverse kinematic problems. The dynamics of the manipulator was modeled using virtual work principle. Expressing the dynamic model in a linear form with respect to inertial parameters enabled us to determine the base inertial parameters and reduce the dynamic model which reduced the computation time by 41%.

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