Kinematics of biplanetary epicyclic gears

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Abstract. In the present paper, the biplanetary gears are analysed. They have complex layouts and their functioning is not too easy to recognize based upon these general schemes. However, it has been proved that two methods i.e. Willis and Kutzbach could be useful for their detailed kinematical analysis, enclosing e.g. directions of rotation of particular geared wheels. The exemplary gears analysed e.g. kinematical ratios have been calculated. The analysis of these gears could be a good training for mechanism understanding by students and researchers.

Key words: biplanetary gear, kinematical gear ratio, Willis equation, scheme of velocities

1 Introduction

Planetary gears are mechanisms consisting – among other – of geared wheels, in which at least the symmetry axis of one wheel encircles the main symmetry axis of the system. The wheel, which is going round, is called a planetary wheel or even just a planet or a satellite gear. In practice, more frequently the mechanical system is a three-shaft-gear which consists of a central geared wheel (sun wheel) 1, an outer ring (annulus) with inward-facing teeth that mesh with the planet gear3and(at least) three planet gears are joined and displaced regularly in the space by means of an arm(carrier) h (Fig. 1a). In this gear, the arm h and two central gearedwheels1 and 3 consists the set of the basic elements. It is described by a code 2WH (2 wheels i.e. W plus an arm H). Depending on fixing or braking of one of its elements, the gear can work as a mechanism of one degree of freedom (1 DOF) however it represents three possible variants of operation. In case when all three basic gear parts are movable, then a planetary gear has two degrees of freedom and there is a demand for driving of two different basic elements.

Therefore, in such a case, a system serves as a differential gear. This case was analyzed by the authors in [6]. Besides Willis method, graph-based approaches were there effectively utilized.

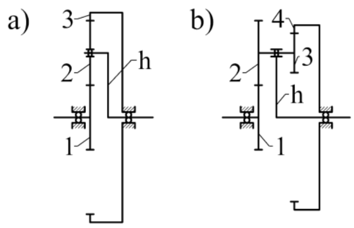

Fig. 1 Schemes of planetary gears - 2WH type (a) oraz 2WH-EI (b)

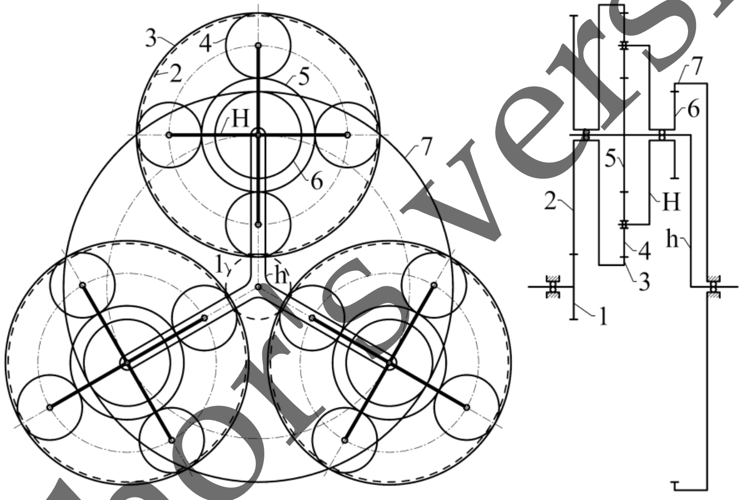

Fig. 2 Front view and scheme of biplanetary gear [5]

In case, when the satellite gears 2 and 3 - for the gear described as 2WH-EI (in Fig.1b) - are connected via an additional kinematic chain, called the internal planetary mechanism. Moreover, we obtain the double planetary gear (biplanetary gear). The biplanetary gears are described e.g.: in books [1,3,5], some of them have been edited recently so there is an interest in this type of mechanisms not only in teaching curricula but also in industry. Namely, just recently it has been described special applications of biplanetary gears [2,7] in power transmissions and chemical equipment. The exemplary scheme of this gear is presented in Fig. 2. It consists of one main planetary gear having the sun wheel 1 geared with the planetary gears 2, the outer arm is denoted by h . Moreover, the system has the second pair of wheels consisting of planetary wheels /number 3 in Fig. 1b/geared with

braked central wheel 7 (number 4 in Fig.1b). The additional kinematic chain is created by the planetary gears 2 and 6 which create an epicyclic gear called as an internal planetary mechanism. It consists of the ring gear 3, planet 4, sun wheel 5 and an inner arm H . The arm H drives the planetary wheel 6 – belonging however to the main gear sub-mechanism. The sun wheel 5, belonging to the inner planetary sub-mechanism, is simultaneously in the considered design variant – the arm h of the main planetary gear.
The value of DoF can be calculated based on the following Kutzbach criterion:

$$W = 3 \cdot n - 2 \cdot p_5 - p_4 = 3 \cdot 5 - 2 \cdot 5 - 4 = 1, \quad (1)$$

where: $n = 5$ - number of moving links, $p_5 = 5$ - number of kinematic pairs of 5-th class (bearings), $p_4 = 4$ - number of kinematic pairs of 4-th class (described as: meshing, geared, in mesh or in gear). Characteristic feature of this gear is that the satellite gears of the epicyclic satellite sub-mechanism performs a complex movement rotating around three axes – own, central of planetary sub-mechanism and central of the main planetary gear. Therefore, the kinematical analysis of the whole system is difficult and complex. These gears – due to the character of their internal movements – are utilized mainly in mine machines (cross-cutters, shearer) as well as agricultural machinery.

2 Kinematic ratio of biplanetary gear

2.1 Calculations performed by means of Willis formula

According to the definition of kinematical ratio, we consider the ratio $i^{7}_{1,h}$ (from pinion 1 to arm h, in case when the wheel 7 is braked /in general immobilized/). The ratio is expressed by means of the following formula:

$$i^{7}_{1,h} = \left(\frac{n_1}{n_h} \right)_{n_7 = 0}, \quad (2)$$

where:
$n_1 = n_{in}$ velocity of pinion 1 - i.e. an input velocity of the considered gear,
$n_h = n_{out}$ velocity of arm h - i.e.an output velocity of the gear (Fig. 2).
Aiming for determination of kinematic ratio $i^{7}_{1,h}$, at the beginning, the basic ratio of the inner gear have to be calculated - in case when the rotational speed is equal to $-n_h$ (i.e. considerations of kinematics in relation to the arm h). In this case, the

relative angular velocities of particular wheels of the gear are equal to: $n_j^h = n_j - n_h$ for $j = 1, 2, \ldots, 7$ (Fig. 2). Similarly, the relative angular velocity of the internal (inner) arm H in relation to the external arm h is equal to: $n_H^h = n_H - n_h$ (Fig. 2).The basic ratio of the inner planetary gear consisting of the wheels 3, 4, 5 and arm H ,in case of known relative velocities: n_3^h, n_4^h, n_5^h and n_H^h - the ratio could be calculated by means of the Willis formula:

$$i_{3,5}^{H} = \frac{n_3^h - n_H^h}{n_5^h - n_H^h} \tag{3}$$

additionally, based on the formulas for ratios related to the set of wheels 3, 4 and 5 in relations to arm H , we obtain the underneath relationship, depending on adequate teeth numbers:

$$i_{3,5}^{H} = \left(\frac{n_3^h - n_H^h}{n_4^h - n_H^h}\right) \cdot \left(\frac{n_4^h - n_H^h}{n_5^h - n_H^h}\right) = \left(\frac{n_3^h - n_H^h}{n_5^h - n_H^h}\right) = \left(-\frac{z_4}{z_3}\right) \cdot \left(-\frac{z_5}{z_4}\right) = \left(\frac{z_5}{z_3}\right), \tag{4}$$

optionally it could be expressed by means of other considered notions:

$$i_{3,5}^{H} = \left(\frac{z_5}{z_3}\right) = \frac{n_3^h - n_H^h}{-n_H^h}, \tag{5}$$

because $n_5^h = n_5 - n_h = 0$ in case when $n_5 = n_h$ (Fig. 2).

Furthermore, aiming for determination of the unknown ratio of the biplanetary gear – the unknown relative angular velocities n_3^h and n_H^h /utilized in the formula (5)/ - have to be determined as functions of: n_h and/or n_1 . It could be done upon two conditions related to the ratio $i_{H,7}$ (i.e. ratio from the arm H to the wheel 7) as well as the ratio $i_{3,1}$ (i.e. ratio from the wheel 3 to the wheel 1):

$$i_{H,7}^{h} = \frac{n_H^h}{n_7^h} = \frac{n_6^h}{n_7^h} = -\frac{z_7}{z_6}, \tag{6}$$

because $n_H = n_6$, therefore additionally we can write $n_H^h = n_H - n_h = n_6 - n_h = n_6^h$, in turn, therefore consecutive formulas can be calculated:

$$n_H^h = n_7^h \cdot \left(-\frac{z_7}{z_6}\right) = n_h \cdot \frac{z_7}{z_6}, \tag{7}$$

because $n_7^h = n_7 - n_h = -n_h$ in case $n_7 = 0$,moreover

$$i^h_{3,1} = \frac{n_3^h}{n_1^h} = \frac{n_2^h}{n_1^h} = -\frac{z_1}{z_2}\,, \tag{8}$$

because $n_3 = n_2$ and the following equalities can be considered as proper for the considered scheme and the assumption made: $n_3^h = n_3 - n_h = n_2 - n_h = n_2^h$. Moreover, taking into account the former considerations–the underneath formula can be obtained:

$$n_3^h = n_1^h \cdot \left(-\frac{z_1}{z_2}\right) = \left(n_1 - n_h\right)\cdot\left(-\frac{z_1}{z_2}\right). \tag{9}$$

Therefore, for the ratio of teeth numbers z_5/z_3 considered in the formula (5) – after some transformations /e.g. considering formula (9)/ - can be expressed by the following relationship:

$$\left(\frac{z_5}{z_3}\right) = \frac{\left(n_1 - n_h\right)\cdot\left(-\frac{z_1}{z_2}\right) - n_h\cdot\frac{z_7}{z_6}}{-n_h\cdot\frac{z_7}{z_6}} = \frac{\left(\frac{n_1}{n_h} - 1\right)\cdot\left(-\frac{z_1}{z_2}\right) - \frac{z_7}{z_6}}{-\frac{z_7}{z_6}}\,, \tag{10}$$

Finally, the formula(2) for determination of the ratio of the biplanetary gear– can be written in the following form:

$$i^7_{1,h} = \left(\frac{n_1}{n_h}\right)_{n_7=0} = 1 - \frac{z_7}{z_6}\cdot\frac{z_2}{z_1}\cdot\left(1 - \frac{z_5}{z_3}\right) = 1 - \frac{-108}{24}\cdot\frac{132}{36}\cdot\left(1 - \frac{42}{-90}\right) = 25.20\,. \tag{11}$$

The calculations were done for the assumed number of teeth for the biplanetary gear: $z_1 = 36$, $z_2 = 132$,gear module $m_{1,2} = 1$, $z_3 = -90$, $z_4 = 24$, $z_5 = 42$, gear module $m_{3,4,5} = 1{,}5$, $z_6 = 24$, $z_7 = -108$, $m_{6,7} = 2$. The notation style has been utilized that teeth number of geared rings are considered as negative.

2.2 Schemes of velocities of particular gear elements

An analysis of tangential velocities of the biplanetary gear could be done independently on the other calculation methods via graphical approach. In the considered case, we introductory assume the value of the rotational velocity ω_h of the arm h (e.g.: $\omega_h = 1\, rad/s$) for the main planetary gear[1,5]. In this case, it is possible to determine the tangential velocity v_h of the arm– via the following formula:

$$v_h = \omega_h \cdot r_h = 1 \cdot 84 \cdot 10^{-3} = 0.084\ m/s\,, \qquad (12)$$

where: r_h -radius of the arm h, in the main planetary gear:

$$r_h = 0.5 \cdot (d_1 + d_2) = 0.5 \cdot m_{1,2} \cdot (z_1 + z_2) = 0.5 \cdot 1 \cdot (36 + 132) = 84\, mm\,. \qquad (13)$$

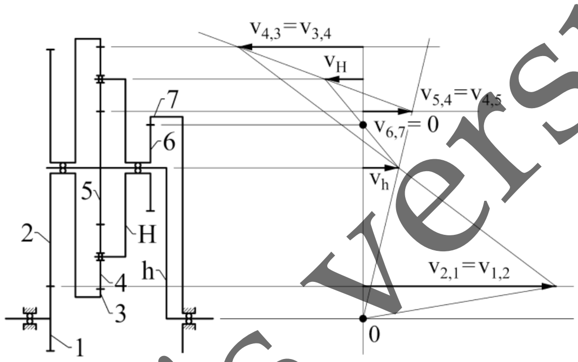

Fig. 3 Scheme of tangential velocities

Vector of this velocity is graphically determined in the way shown in (Fig. 3). Point of gearing (meshing) of the satellite 6 with the immobilized wheel 7 is the temporary central point of rotation of the wheel 6; i.e. the tangential velocity of the wheel 6 in this point is equal to $v_{6,7} = 0$ .Therefore, the vector v_H (i.e. velocity of the arm H) is determined by the straight line going from the end of the vector v_h via the point relevant to the zero velocity of the wheel 6, in the temporary center of rotational movement. Its value is derived from the proportion:

$$\frac{v_h}{r_6} = \frac{v_H}{r_H - r_6}\ ;\ v_H = v_h \cdot \frac{r_H - r_6}{r_6} = 0.084 \cdot \frac{49.5 - 24}{24} = 0.08925\, m/s \qquad (14)$$

where: r_H - radius of the external arm H and r_6 - pitch radius of the satellite 6:

$$r_h = 0.5 \cdot (d_5 + d_4) = 0.5 \cdot m_{3,4,5} \cdot (z_5 + z_4) = 0.5 \cdot 1.5 \cdot (42 + 24) = 49.5\, mm\,, \qquad (15)$$

$$r_6 = 0.5 \cdot m_{6,7} \cdot z_6 = 0.5 \cdot 2 \cdot 24 = 24\ mm\,. \qquad (16)$$

The sun wheel 5 creates a common element with the arm h, therefore the vector of velocity $v_{5,4}$ of the sun wheel 5 in the point of meshing with the satellite 4 is determined by the straight line going from the point O i.e. the center of rotation of the arm h via the endpoint of the velocity vector v_h. The value of velocity $v_{5,4} = v_{4,5}$ can be calculated upon the proportion:

$$\frac{v_h}{r_h} = \frac{v_{5,4}}{r_h + r_5};\ v_{5,4} = v_h \cdot \frac{r_h + r_5}{r_h} = 0.084 \cdot \frac{84 + 31.5}{84} = 0.1155\ m/s \qquad (17)$$

where: r_5 - pitch radius r_5 of the sun wheel 5:

$$r_5 = 0.5 \cdot m_{3,4,5} \cdot z_5 = 0.5 \cdot 1.5 \cdot 42 = 31.5\ mm\,. \qquad (18)$$

Knowing the velocity $v_{4,5} = v_{5,4}$ of the satellite 4 in the touch point with the sun wheel 5 and velocity v_H of the central point of the satellite 4;- it is possible to establish velocity $v_{4,3}$ of the satellite 4 in the touch point with the ring gear 3. In consequence, the value of vector $v_{4,5}$ is analytically determined based upon the proportion:

$$\frac{v_{4,5} + v_H}{r_4} = \frac{v_{4,3} - v_H}{r_4}\,, \qquad (19)$$

therefore:

$$v_{4,3} = v_{4,5} + 2 \cdot v_H = 0.1155 + 2 \cdot 0.08925 = 0.294\ m/s\,, \qquad (20)$$

where: $r_4 = 0.5 \cdot m_{3,4,5} \cdot z_4 = 0.5 \cdot 1{,}5 \cdot 24 = 18\ mm$ - pitch radius of the satellite wheel 4.

Based on relationships form vector analysis $v_{3,4} = v_{4,3}$ and $v_{2,1} = v_{1,2}$ we have;

$$\frac{v_{2,1} - v_h}{r_2} = \frac{v_{3,4} + v_h}{|r_3|}\,, \qquad (21)$$

we have the formula:

$$v_{2,1} = \left(v_{3,4} + v_h\right) \cdot \frac{r_2}{|r_3|} + v_h = \left(0.294 + 0.084\right) \cdot \frac{66}{|-67.5|} + 0.084 = 0.4536\ m/s\,, \qquad (22)$$

where: r_2, r_3 - pitch radiuses of the satellite wheel 2 and the ring 3, respectively:

$$r_2 = 0.5 \cdot m_{1,2} \cdot z_2 = 0.5 \cdot 1 \cdot 132 = 66\ mm\,, \qquad (23)$$

$$r_3 = 0.5 \cdot m_{3,4,5} \cdot z_3 = 0.5 \cdot 1{,}5 \cdot \left(-90\right) = -67.5\ mm\,. \qquad (24)$$

Finally, the searched (unknown) angular velocity ω_1 of the sun wheel 1,for the assumed angular velocity $\omega_h = 1\, rad/s$ of the external (outer) arm H , is equal to:

$$\omega_1 = \frac{v_{1,2}}{r_1} = \frac{0.4536}{18.5} \cdot 10^3 = 25.20\, rad/s\,, \qquad (25)$$

where: $r_1 = 18\, mm$ -pitch radius of pinion (i.e. sun wheel).
Finally it was obtained the same value of kinematic ratio:

$$i^7_{1,h} = \left(\frac{\omega_1}{\omega_h}\right)_{\omega_7 = 0} = \frac{25.20}{1} = 25.20\,. \qquad (26)$$

The analysis was performed for an exemplary gear but the applied approach could be used in analysis of these types of planetary gear, in general.

3 Conclusions

The discussed methods i.e.: Willis's and Kutzbach's belong to the most effective and general approaches. Therefore, they are useful for an analysis of velocities (kinematics) of biplanetary gears which are rarely considered, despite the applications in control and chemical devices. Based on the utilized methodology, it is not only possible to established velocities of particular parts but also one can calculate the kinematic ratio(s) as well as directions of rotation of all wheels. Furthermore, these approaches are useful – in general, for other complex planetary gears [6].

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