Modeling and Kinematic Nonlinear Control of Aerial Mobile Manipulators

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Abstract. This work proposes a kinematic modeling and a kinematic nonlinear controller for an autonomous aerial mobile manipulator robot that generates saturated reference velocity commands for trajectory tracking problem. In the kinematic modeling is considered through of a quadcopter-inner-loop system to independently track four velocity commands: forward, lateral, up/downward, and heading rate; and arm-inner-loop system to independently track angular velocity commands. Stability and robustness of the complete control system are proved through the Lyapunov method. Finally, simulation results are presented and discussed, which validate the proposed controller.

Key words: Aerial Mobile Manipulators, Kinematic Modeling, Nonlinear Controller, Lyapunov.

1 Introduction

The area of Robotics has evolved presenting new technologies that allow to improve the intelligence and mobility of robots. Mobile manipulators have been one of the main topics of academic research in recent years and allowing more sophisticated tasks, especially for unmanned aerial vehicles (UAVs), the mobility of these is not limited to displacement on flat surfaces, Expanding tasks such as: *i)* construction of high platforms [1]; *ii)* cargo transport to unaffordable areas [2]; *iii)* aplicaciones en lineas de alta tension [3]; *iv)* tasks that are dangerous or monotonous to humans, among others.[4],[5].

For the mobility of the robots, platforms have been developed that can work in environments: terrestrial, aquatic and air, for this are used wheels / legs, propellers and propellers [6] [7] [8]. The combination of mobile platforms with robotic arms are denominated as mobile manipulators, these allow to increase the workspace and applications in the domestic, commercial, mititar area, among others. There are several ways of performing the study and control of these systems, i) one of them is to do it separately, *i.e.,* the kinetic model is made of the mobile platform as the manipulator, also the control is made to each of these parts, The point of interest of the kinematic analysis of the mobile platform is done with respect to the center of mass and the point of interest of the kinetic analysis of the manipulator is done with respect to the operating end; ii) the kinematic study is done together, *i.e.,* kinematic modeling and control is done from the system together, for the modeling and control of this system is done with respect to the end effector of the mobile manipulator.

This paper presents a non-linear control strategy for resolving the trajectory tracking problem of a aerial mobile maipulator. Which is constituted by an quadcopter mounting a robotic arm of 3 degrees of freedom mounted on back of base. For the design of the controller, the kinematic model of the aerial mobile manipulator is used which has as input the velocity and orientation, this controller is designed based on seven velocities commands of the aerial mobile manipulator, four corresponding to the aerial platform: forward, lateral, up/downward and orientation, the last three are those who command the manipulator robot. It is also pointed out that the workspace has a single reference that is located in the operative end of the aerial mobile manipulator $< R(x\ y\ \mathrm{z}) >$. The stability of the controller is analyzed by the Lyapunov´s method and to validate the proposed control algorithm, experimental processes are presented and discussed in this paper.

2 Aerial Mobile Manipulators Model

The mobile manipulator configuration is defined by a vector $\mathbf{q}$ of n independent coordinates, called generalized coordinates of the aerial mobile manipulator, where $\mathbf{q} = \begin{bmatrix} q_1 & q_2 & \dots & q_n \end{bmatrix}^T = \begin{bmatrix} \mathbf{q}_\mathbf{h}^T & \mathbf{q}_\mathbf{a}^T \end{bmatrix}^T$ where $\mathbf{q}_\mathbf{a}$ represents the generalized coordinates of the *robotic arm*, and $\mathbf{q}_\mathbf{h}$ the generalized coordinates of the *aerial mobile platform*. We notice that $n = n_h + n_a$, where n_h and n_a are respectively the dimensions of the generalized spaces associated to the aerial mobile and to the robotic arm. The configuration $\mathbf{q}$ is an element of the aerial mobile manipulator *configuration space*; denoted by $\mathcal{N}$. The location of the end-effector of the aerial mobile manipulator is given by the m–dimensional vector $\mathbf{h} = \begin{bmatrix} h_1 & h_2 & \dots & h_m \end{bmatrix}^T$, where $\mathbf{h}$ define the position and the orientation, respectively, of the end-effector of the aerial mobile manipulator in $\mathcal{R}$. Its m coordinates are the *operational coordinates of the aerial mobile manipulator*. The set of all locations constitutes the *aerial mobile manipulator operational space*, denoted by $\mathcal{M}$.

The location of the aerial mobile manipulator end-effector can be defined in different ways according to the task, *i.e.*, it can be considered only the position of the end-effector or both its position and its orientation.

2.1. Aerial Mobile Manipulator Kinematic Modeling

The *kinematic model of an aerial mobile manipulator* gives the location of the end-effector $\mathbf{h}$ as a function of the robotic arm configuration and the aerial mobile location (or its operational coordinates as functions of the robotic arm's generalized coordinates and the mobile aerial's operational coordinates).

$$f: \ \mathcal{N}_a \times \mathcal{M}_{\hbar} \ \rightarrow \ \mathcal{M}$$
$$(\mathbf{q_h}, \mathbf{q_a}) \ \mapsto \ \mathbf{h} = f(\mathbf{q_h}, \mathbf{q_a})$$

where, $\mathcal{N}_a$ is the *configuration space* of the robotic arm, $\mathcal{M}_{\hbar}$ is the *operational space of the aerial mobile*.

The *instantaneous kinematic model of an aerial mobile manipulator* gives the derivative of its end-effector location as a function of the derivatives of both the robotic arm configuration and the location of the aerial mobile platform,

$$\dot{\mathbf{h}} = \frac{\partial f}{\partial \mathbf{q}}(\mathbf{q_p}, \mathbf{q_a})\mathbf{v}$$

where, $\dot{\mathbf{h}} = [\dot{h}_1 \quad \dot{h}_2 \quad ... \quad \dot{h}_m]^T$ is the vector of the end-effector velocity, $\mathbf{v} = [v_1 \quad v_2 \quad ... \quad v_{\delta_n}]^T = [\mathbf{v_h}^T \quad \mathbf{v_a}^T]^T$ is the control vector of mobility of the aerial mobile manipulator. Its dimension is $\delta_n = \delta_{nh} + \delta_{na}$, where δ_{nh} and δ_{na} are respectively the dimensions of the control vector of mobility associated to the aerial mobile platform and the robotic arm, respectively.

Now, after replacing $\mathbf{J}(\mathbf{q}) = \frac{\partial f}{\partial \mathbf{q}}(\mathbf{q_h}, \mathbf{q_a})$ in the above equation, we obtain

$$\dot{\mathbf{h}}(t) = \mathbf{J}(\mathbf{q})\mathbf{v}(t) \tag{1}$$

where, $\mathbf{J}(\mathbf{q})$ is the Jacobian matrix that defines a linear mapping between the vector of the aerial mobile manipulator velocities $\mathbf{v}(t)$ and the vector of the end-effector velocity $\dot{\mathbf{h}}(t)$. The Jacobian matrix is, in general, a function of the configuration $\mathbf{q}(t)$.

2.2. Case Study: Quadcopter and Robotic Arm

The kinematic model of the aerial mobile manipulator is composed by a set of seven velocities represented at the spatial frame $< H >$. The displacement of the aerial mobile manipulator is guided by the three linear velocities u_l, u_m and u_n defined in a rotating right-handed spatial frame $< H >$, and the angular velocity ω, as shown in Fig. 1.

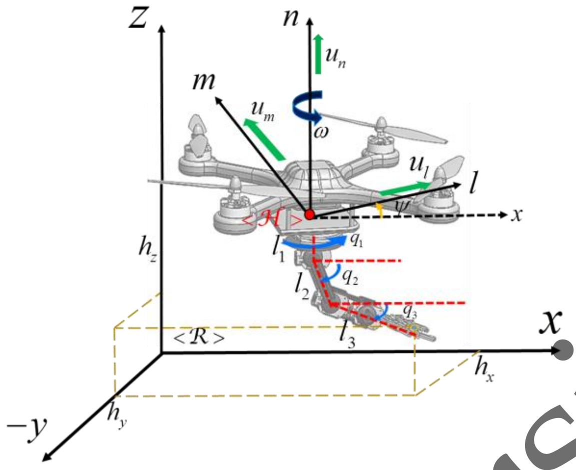

Fig. 1 Schematic of the aerial mobile manipulator.

Each linear velocity is directed as one of the axes of the frame $<H>$ attached to the center of gravity of the quadcopter: u_l points to the frontal direction; u_m points to the left-lateral direction¸ and u_n points up. The angular velocity ω rotates the referential system $<H>$ counterclockwise, around the axis H_Z (considering the top view). While the maneuverability of the robotic arm is defined by three angulars velocities with respect to the reference system $<H>$, *i.e.*, $\dot{q}_1$ rotates with respect to the axis n, and $\dot{q}_2$, $\dot{q}_3$ rotate with respect to the axis m of the reference system $<H>$. In other words, the Cartesian motion of the aerial mobile manipulator at the inertial frame $<\mathcal{R}>$ is defined as,

$$\begin{cases} \dot{h}_x = u_l C_\psi - u_m S_\psi + l_2 S_{q2} C_{\psi q1} \dot{q}_2 + l_2 C_{q2} S_{\psi q1}(\dot{\psi} + \dot{q}_1) + l_3 S_{q2q3} C_{\psi q1}(\dot{q}_2 + \dot{q}_3) + l_3 C_{q2q3} S_{\psi q1}(\dot{\psi} + \dot{q}_1) \\ \dot{h}_y = u_l S_\psi - u_m C_\psi + l_2 S_{q_2} S_{\psi q_2} \dot{q}_2 - l_2 C_{q_2} C_{\psi q1}(\dot{\psi} + \dot{q}_1) + l_3 S_{q_2 q_3} S_{\psi q1}(\dot{q}_2 + \dot{q}_3) - l_3 C_{q_2 q_3} C_{\psi q1}(\dot{\psi} + \dot{q}_1) \\ \dot{h}_z = u_n - l_1 - l_2 S_{q_2} - l_3 S_{q_2 q_3} \end{cases} \quad (2)$$

where h_x, h_y h_z and ψ are all measured with respect to the inertial frame $<\mathcal{R}>$; $C_\alpha = \cos(\alpha)$; $C_{\alpha\beta} = \cos(\alpha+\beta)$; $S_\alpha = \sin(\alpha)$ and $S_{\alpha\beta} = \sin(\alpha+\beta)$. The point of interest (whose position is being controlled) is the end/effector of the aereal mobile manipulator. Also the equation system (2) can be written in compact form as $\dot{\mathbf{h}} = f(\mathbf{h}, \mathbf{q})\mathbf{u}$, *i.e.*,

$$\dot{\mathbf{h}}(t) = \mathbf{J}(\mathbf{q}, \psi)\mathbf{v}(t) \quad (3)$$

where, $\mathbf{J}(\mathbf{q}, \psi) \in \Re^{mxn}$ with $m = 3$ and $n = 7$ represents the Jacobian matrix that defines a linear mapping between the velocity vector of the aerial mobile manipulator $\mathbf{v} \in \Re^n$ where $\mathbf{v} = [u_l \ u_m \ u_n \ \psi \ \dot{q}_1 \ \dot{q}_2 \ \dot{q}_3]^T$ and the velocity vector of the operative end $\dot{\mathbf{h}} \in \Re^m$ where $\dot{\mathbf{h}} = [\dot{h}_x \ \dot{h}_y \ \dot{h}_z]^T$.

3 Controller Design and Stability Analysis

As represented in Fig.2 , the trajectory is given time-varying trajectory $\mathbf{h_d}(t)$ and it's successive derivatives $\dot{\mathbf{h}}_\mathbf{d}(t)$ which respectively describe the desired velocity of the robot. That's, the desired trajectory for the end-effector of the aerial mobile manipulator is defined by a vector $\mathbf{h_d}(t) = [h_{xd} \quad h_{yd} \quad h_{zd}]^T$ in $< \mathcal{R}(\mathcal{X}, \mathcal{Y}, \mathcal{Z}) >$. The desired trajectory doesn't depend on the instantaneous position of the end-effector of the aerial mobile manipulator, but it's defined only by the time varying trajectory profile alone.

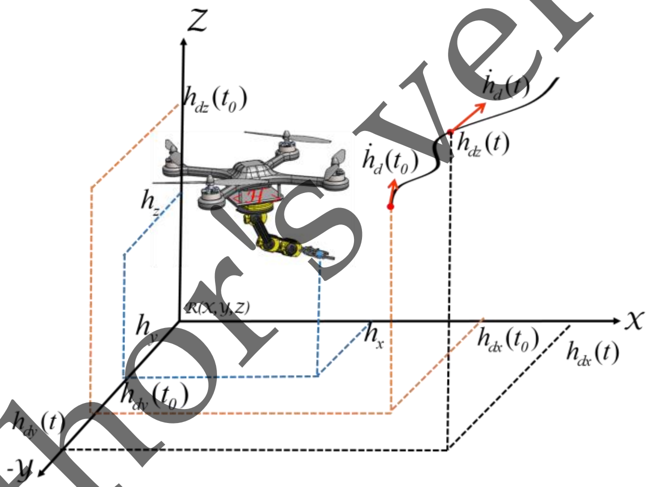

Fig. 2 Problem of control of the trajectory tracking.

The controller proposed to solve the trajectory tracking problem of the aerial mobile manipulator, the proposed kinematic controller is based on the kinematic model of the aerial mobile manipulator (3). Hence following control law is proposed,

$$\mathbf{v} = \mathbf{J}^{\#}\left(\dot{\mathbf{h}}_\mathbf{d} + \mathbf{L_K} \tanh\left(\mathbf{L_K^{-1}} \mathbf{K} \, \tilde{\mathbf{h}}\right)\right) \tag{4}$$

where $\dot{\mathbf{h}}_\mathbf{d}$ is the velocity of the aerial mobile manipulator for the controller; $\mathbf{J}^{\#}$ is the matrix of pseudoinverse kinematics for the aerial mobile manipulator where $\mathbf{J}^{\#} = \mathbf{W}^{-1}\mathbf{J}^\mathbf{T}\left(\mathbf{J}\mathbf{W}^{-1}\mathbf{J}^\mathbf{T}\right)^{-1}$ with $\mathbf{W}$ being a definite positive matrix that weighs the control actions of the system; while that $l_x > 0, k_x > 0, l_y > 0$, $k_y > 0$, $l_z > 0$ and

$k_z > 0$ area gain constants of the controller that weigh the control error respect to the inertial frame $<\mathcal{R}>$; and the **tanh(.)** represents the function saturation of maniobrability velocities in the aerial mobile manipulator.

The other hand, the behaviour of the control error of the end-effector $\mathbf{h}(t)$ is now analyzed assuming perfect velocity tracking. By substituting (4) in (3) it is obtained the close loop equation,

$$\dot{\mathbf{h}}_{\mathbf{d}} + \mathbf{L}\tanh\left(\tilde{\mathbf{h}}\right) = \mathbf{0} \tag{5}$$

For the stability analysis the following Lyapunov candidate function is considered

$$V\left(\tilde{\mathbf{h}}\right) = \tfrac{1}{2}\tilde{\mathbf{h}}^{\mathbf{T}}\tilde{\mathbf{h}}\,. \tag{6}$$

Its time derivative on the trajectories of the system is, $\dot{V}\left(\tilde{\mathbf{h}}\right) = -\tilde{\mathbf{h}}^{\mathbf{T}}\mathbf{L}_{\mathbf{K}}\tanh\left(\mathbf{L}_{\mathbf{K}}^{\mathbf{-1}}\mathbf{K}\,\tilde{\mathbf{h}}\right)$. A sufficient condition for $\dot{V}\left(\tilde{\mathbf{h}}\right)<0$ to be negative definite is,

$$\tilde{\mathbf{h}}^{\mathbf{T}}\mathbf{L}_{\mathbf{K}}\tanh\left(\mathbf{L}_{\mathbf{K}}^{\mathbf{-1}}\mathbf{K}\,\tilde{\mathbf{h}}\right) > 0 \tag{7}$$

Hence, according to (4) and recalling that $\mathbf{K}$ is diagonal positive definite, the control error vector $\lim_{t\to 0}\tilde{\mathbf{h}}(t) = 0$ asymptotically.

4 Results and Discussions

This Section presents the simulation results of the waypoint tracking flight task in the 3D space using the kinematic nonlinear controller designed in the previous section. The goal of the simulations is to test the stability and performance of the proposed controller. Fig. 4 represents the block diagram of the simulation system. The quadcopter model considers not-ideal dynamics, such as flapping, drag, and actuator dynamics, and it describes accurately the system's dynamics both for hovering and for low speed translational flights.

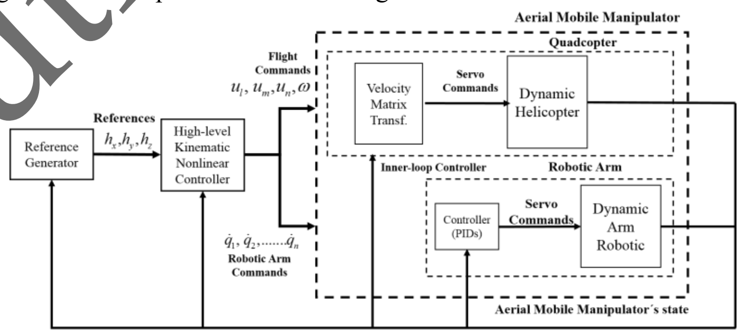

Fig. 4 Block diagram of the simulation system.

In order to assess and discuss the performance of the proposed controller, it was developed a simulation platform for aerial mobile manipulators with Matlab interface, see the Fig. 5. This is an online simulator, which allows users to view three-dimensional environment navigation of the robot.

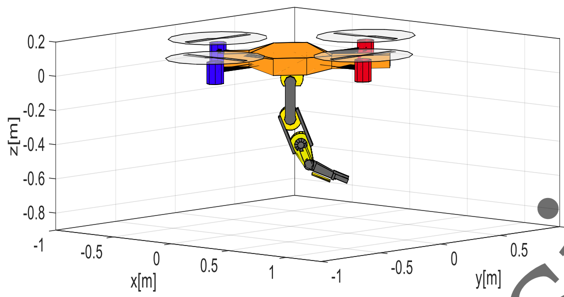

Fig. 5 Aerial mobile manipulator robot used by simulation platform developed

For the simulation presented below the tajetory tracking to be followed is a saddle described by, $h_{xd}=0.07t$; $h_{xd}=0.2+0.7\sin(0.2t)$ and $h_{xz}=3+0.2\sin(0.4t)$. The desired velocity of the end-effector of the aerial mobile manipulator will depend of the desired traejectory. Fig. 6 shows the stroboscopic movement on the $\mathcal{X}$-$\mathcal{Y}$-$\mathcal{Z}$ respect to the inertial frame $<\mathcal{R}>$. It can be seen that the proposed controller works correctly. The position error $\tilde{h}_x,\tilde{h}_y,\tilde{h}_z$ of the aerial mobile manipulator is illustrated in Fig. 7, where it can seen the error $\tilde{\mathbf{h}}(t)\rightarrow\mathbf{0}$ asymptotically.

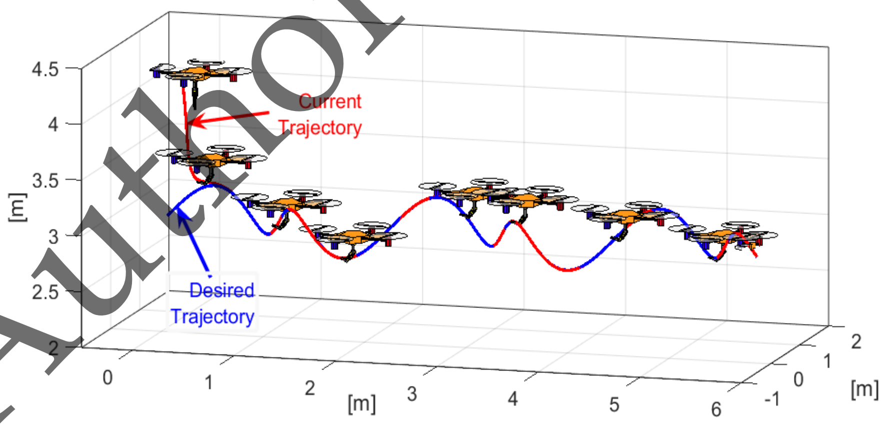

Fig. 6 Stroboscopic movement of the aerial mobile manipulator in the trayectory tracking problem.

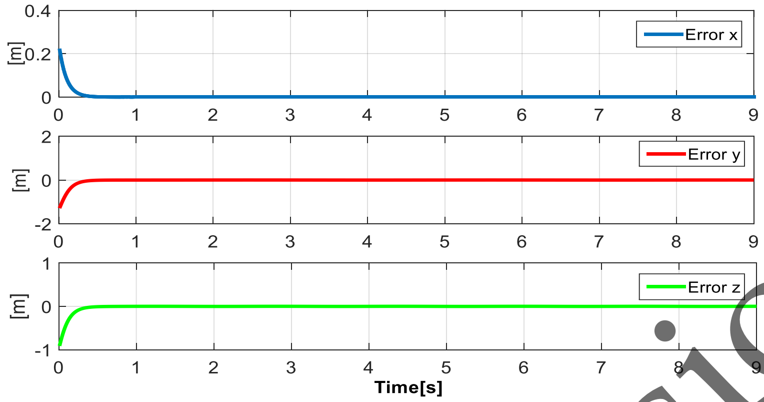

Fig. 7 Control errors of the end-effector of the aereal mobile manipulator.

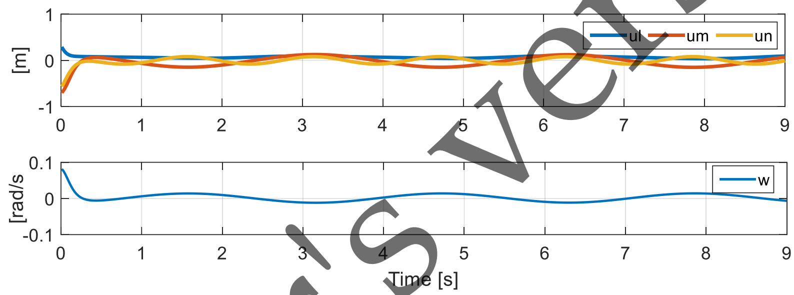

Fig. 8 Commands of Drone reference velocities by controller.

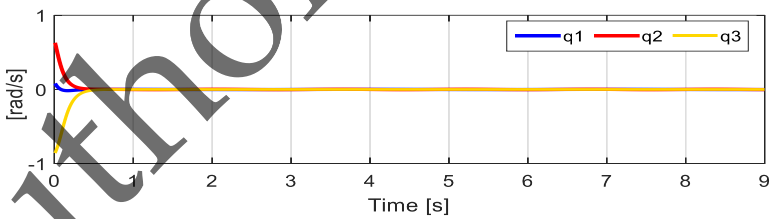

Fig. 9 Commands of Arm reference velocities by controller.

5 Conclusions

A kinematic controller -responsible to accomplish the task of trajectory tracking- is here proposed to solve the 3D trajectory tracking problem for a miniature aerial mobile manipulator. The main advantage of the control laws here proposed lies in their simplicity and easiness of implementation, when compared to other yet available in the literature. In addition, the system stability has been analytically proven. The simulation's results have proven the controller's ability to globally and asymptotically drive the controlled state variables to zero and simutaneously prevent any saturation in the flight commands. As future work, the implementation of such control system onboard a real aerial mobile manipulator will be tested, whose results are expected to confirm the effectiveness of the proposed control system.

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