Evaluating the knot vector to synthesize the cam motion using NURBS

T.T.N. Nguyen[1], S. Kurtenbach[2], M. Hüsing[3] and B. Corves[4]

Department of Mechanism Theory and Dynamics of Machines,
RWTH Aachen University, Germany
[1] *e-mail: Nguyen-Thi@igm.rwth-aachen.de*
[2] *e-mail: kurtenbach@igm.rwth-aachen.de*
[3] *e-mail: huesing@igm.rwth-aachen.de*
[4] *e-mail: corves@igm.rwth-aachen.de*

Abstract. A Non Uniform Rational B-Spline (NURBS) is used for synthesizing the motion curve of cam mechanisms because it is flexible and satisfies arbitrary boundary conditions from the working requirement of machinery systems. For using NURBS curve as motion curves of cam mechanisms, selecting the knot vector is very important. This work presents the effect of the knot vector on the displacement, velocity, acceleration, and jerk curves. The linear system of equations for solving the cam motion is also presented. A general computation of the knot vector of NURBS for synthesizing the motion curve is presented. Several examples illustrate this research.

Key words: Parameterization method, knot vector, NURBS, cam motion synthesis.

1 Introduction

The synthesis of motion curves for cam mechanisms depends on working requirements and application situations of machinery systems. The boundary conditions of the displacement function are not only displacement constraints but also the velocity, acceleration, and jerk constraints. Frequently, designers must refine the displacement functions, where their derivatives can reduce the maximum values of acceleration and jerk.

There are several standard functions such as harmonic, cycloidal, trapezoidal, and polynomial [1-3]. The disadvantage of these functions is limited by a number of boundary conditions. For motion curves, polynomial functions are commonly used in cam design. However, the displacement curves can be oscillating with high order of polynomial when the number of boundary conditions becomes large. Therefore, acceleration and jerk curves can occur the peak values.

In several recent decades, spline functions, B-spline and NURBS curves, have been used to synthesize motion curves of cam mechanisms [8-13]. The main advantage of using these curves for displacement functions is unlimited boundary conditions from working requirements. Moreover, these curves and their derivatives can be controlled by several parameters such as the knot vector, control points, and weights. The knot vector is one of the important parameters since it is directly connected with the shape of these curves. The uniform spacing method is commonly used for calculating the knot vector as shown in [8-11]. This method is very comfortable to calculate the knot vector. Other researches used the knot vector that is specified in the increasing direction of the independent cam rotation [12, 13].

Until now, the knot vector is still interesting to calculate the shape of curves. In this paper, we present the effect of the knot vector on the kinematics of the cam motion. Several methods for computing the knot vector are used for NURBS curve. Here, the study cases with a large number of boundary conditions of the displacement, velocity, acceleration, and jerk are considered to synthesize of the motion curve of cam mechanisms.

The organization of this paper is as follows. Section 2 shows the description of NURBS and briefly presents a general synthesis of motion curves with NURBS. The linear system of equations is established as follows. The computation of knot vector for synthesizing motion curves is present in section 3. Section 4 shows the effect of knot vector to motion curves by several examples. Conclusion is presented in section 5.

2 Description of NURBS curve for cam motion

2.1 NURBS curve formulation

A detailed introduction to Non-Uniform Rational B-Spline (NURBS) curve can be found in [4]. The NURBS curve of degree, p, is defined by $n+1$ control points P_i, $i = 0, ..., n$ and knot vector $\mathbf{U}$. The NURBS curve is expressed as

$$C(u) = \frac{\sum_{i=0}^{n} N_{i,p}(u)\, w_i P_i}{\sum_{j=0}^{n} N_{j,p}(u)\, w_j}, \quad u \in [a,b]. \tag{1}$$

Here, w_i are weights and they are positive. $N_{i,p}$ are the B-spline basis functions that are defined over the knot vector $\mathbf{U}$

$$\mathbf{U} = \{u_0, u_1, u_2, ..., u_m\}, \tag{2}$$

with $m = n + p + 1$. The knot vector is a nondecreasing sequence of real number and u_i are called knots. The knot vector is also expressed as

$$\mathbf{U} = \{\underbrace{a, ..., a}_{p+1}, u_{p+1}, ..., u_{m-p-1}, \underbrace{b, ..., b}_{p+1}\}. \tag{3}$$

From Eq. (1), the basis functions $N_{i,p}$ are calculated by using the knot vector as

$$N_{i,0}(u) = \begin{cases} 1 & \text{for} \quad u_i \leq u < u_{i+1} \\ 0 & \text{otherwise} \end{cases}$$
$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u). \tag{4}$$

Setting

$$R_{i,p}(u) = \frac{N_{i,p}(u)\, w_i}{\sum_{j=0}^{n} N_{j,p}(u)\, w_j}, \tag{5}$$

they are called the rational basis functions. Thus, the Eq. (1) can be written as

$$C(u) = \sum_{i=0}^{n} R_{i,p}(u)\, P_i. \tag{6}$$

Furthermore, the k^{th} derivative of NURBS curve can be computed as

$$C^k(u) = \sum_{i=0}^{n} R_{i,p}^k(u)\, P_i. \tag{7}$$

2.2 Cam motion using NURBS curve

The derivative of NURBS curve with degree p is continuous up to $(p-1)$. Therefore, in this paper, the NURBS curve with degree $p = 5$ is used for synthesizing the motion curves because its derivative is continuous up to jerk function.

With the cam motion using NURBS curve, we denote u as the angle of cam shaft. The given boundary conditions of the displacement, velocity, acceleration, and jerk are respectively $C(u_j)$, $C^1(u_k)$, $C^2(u_l)$, and $C^3(u_h)$ at u_j, u_k, u_l, and u_h. For the number of boundary conditions, $n+1 = d+e+f+g$, the linear system of equations can be written as

$$\mathbf{C} = \mathbf{R}\mathbf{P}, \tag{8}$$

where the matrix $\mathbf{C}$ with size $(n+1) \times 1$ can be expressed by

$$\mathbf{C} = \begin{bmatrix} C(u_j) & C^1(u_k) & C^2(u_l) & C^3(u_h) \end{bmatrix}^T, \tag{9}$$

for $j = 1, \ldots, d$, $k = 1, \ldots, e$, $l = 1, \ldots, f$, and $h = 1, \ldots, g$.

Here, the matrix $\mathbf{R}$ with size, $(n+1) \times (n+1)$, presents the values of rational basis functions, the first derivative, the second derivative, and the third derivative at u_j, u_k, u_l, and u_h respectively. $\mathbf{R}$ can be written as

$$\mathbf{R} = \begin{bmatrix} R_{i,p}(u_j) & R_{i,p}^1(u_k) & R_{i,p}^2(u_l) & R_{i,p}^3(u_h) \end{bmatrix}^T, \quad \text{for} \quad i = 0, \ldots, n. \tag{10}$$

From Eq. (8), $\mathbf{P}$ can be presented by

$$\mathbf{P} = \left[P_0, P_1, ..., P_n\right]^T . \quad (11)$$

As mentioned above, P_i are control points.

Solving the linear system of equations as shown in Eq. (8), we obtain the control points. It means that the motion curve of cam mechanisms is established.

3 Computation of the knot vector for synthesizing cam motion

3.1 Parameterization method to generate the knot vector

According to Eq. (5), the basis functions are established by the knot vector $\mathbf{U}$ as shown in Eq. (3). For the number of boundary conditions at u_j, u_k, u_l, and u_h, the input angle vector of camshaft, denoted by $\mathbf{D}$, is arranged from small to big value of the cam rotation in the order u_j, u_k, u_l, and u_h. Thus, for $n+1$ input angles, $\mathbf{D}$ can be written as

$$\mathbf{D} = [D_0, D_1, ..., D_n] . \quad (12)$$

The vector, denoted by $\mathbf{t} = [t_0, t_1, ..., t_n]$, has $n+1$ parameters. To compute these parameters, we present three methods such as uniformly space method, chord length method, and centripetal method. From Eq. (1), the angle of camshaft as u is in the parameter domain $[a, b]$, with $a = D_0$ and $b = D_n$.

The uniformly space method has been presented in [6]. With the end parameters $t_0 = a$ and $t_n = b$, the remaining parameters are computed by

$$\begin{cases} t_0 = a \\ t_i = a + i\dfrac{b-a}{n} \qquad \text{for} \quad i = 1, ..., n-1. \\ t_n = b \end{cases} \quad (13)$$

The detail of chord length parameterization method can be found in [14]. The end parameters are $t_0 = a$ and $t_n = b$. The other parameters are calculated by

$$\begin{cases} t_0 = a \\ t_i = a + \dfrac{\sum\limits_{i=1}^{k} |D_i - D_{i-1}|}{\sum\limits_{i=1}^{n} |D_i - D_{i-1}|}(b-a) \qquad \text{for} \quad k = 1, ..., n-1. \\ t_n = b \end{cases} \quad (14)$$

Respectively, the centripetal parameterization method [7] at the first and the end parameters are $t_0 = a$ and $t_n = b$. The remaining parameters are expressed as

$$\begin{cases} t_0 = a \\ t_i = a + \dfrac{\sum_{i=1}^{k} |D_i - D_{i-1}|^{\alpha}}{\sum_{i=1}^{n} |D_i - D_{i-1}|^{\alpha}} (b-a) \quad \text{for} \quad k = 1, ..., n-1 \ , \\ t_n = b \end{cases} \tag{15}$$

with the positive power as α is in $[0,1]$. Selecting the value α affects the shape of the displacement, velocity, acceleration, and jerk curves. In this paper, we do not discuss the effect of the parameter α. For calculating the parameters t_i according to the centripetal method, we choose the value $\alpha = 1/2$ that is the square root of chord length method.

3.2 Knot vector generation

To generate the knot vector for NURBS after a set of parameters t_i is obtained. Using NURBS with degree p for motion curves, we need to compute $m+1$ knots from $n+1$ parameters in $\mathbf{t}$, where $m = n+p+1$. According to the knot vector in Eq. (3), we have $p+1$ knots with $u_0 = u_1 = ... = u_p = a$ and $u_{m-p} = u_{m-p+1} = ... = u_m = b$. The remaining $n-p$ interval knots $(u_{p+1}, ..., u_{m-p-1})$ are computed from the parameters t_i.

The uniformly spaced knot vector can be calculated by [6]

$$\begin{cases} u_0 = u_1 = ... = u_p = a \\ u_{j+p} = t_0 + \dfrac{j}{n-p+1} (b-a) \quad \text{for} \quad j = 1, 2, ..., n-p. \\ u_{m-p} = u_{m-p+1} = ... = u_m = b \end{cases} \tag{16}$$

For the chord length and the centripetal method, the knot vector is computed by the average method [2]

$$\begin{cases} u_0 = u_1 = ... = u_p = a \\ u_{j+p} = t_0 + \dfrac{1}{p} \sum_{i=j}^{j+p-1} t_i \quad \text{for} \quad j = 1, 2, ..., n-p. \\ u_{m-p} = u_{m-p+1} = ... = u_m = b \end{cases} \tag{17}$$

4 Results and discussions

This section presents two examples with a large number of boundary conditions. In the first example, the follower of the cam mechanism satisfies 20 boundary conditions (see in [8]) of the displacement, velocity, and acceleration as shown by start signs in Fig. 1. From the given angles of camshaft, the input angle vector is expressed as $\mathbf{D} = [0,\ 0,\ 0,\ 0.7854,\ 0.7854,\ 1.5708,\ 1.5708,\ 1.5708, 2.3562,\ 2.3562,\ 2.6180,\ 3.1416,\ 3.1416,\ 3.1416,\ 3.6652,\ 3.9270,\ 3.9270,\ 4.7124, 4.7124,\ 4.7124]$. The knot vectors for the uniform space, chord length and centripetal methods are computed in section 3.

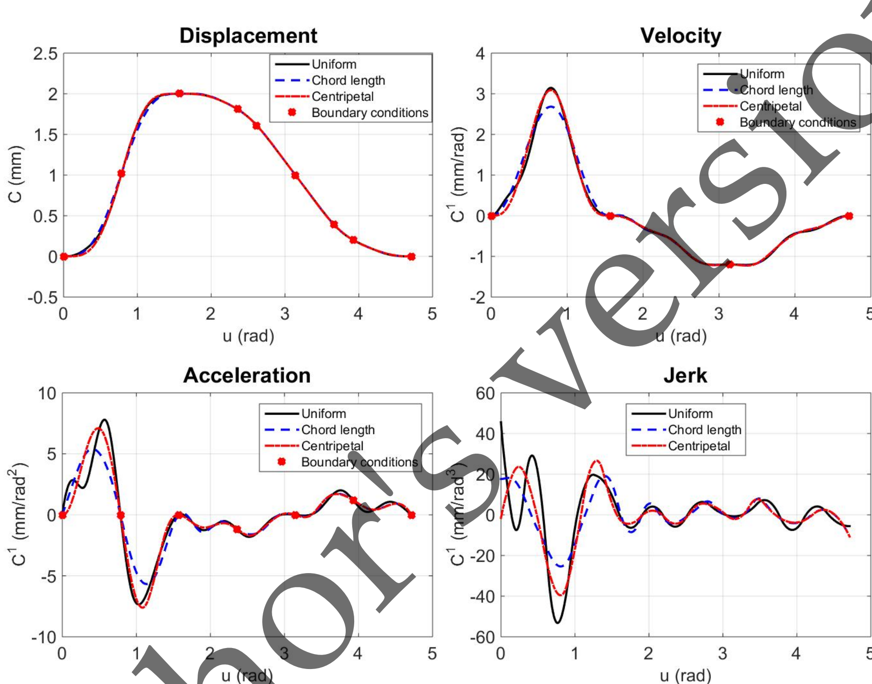

Fig. 1 Comparison of motion curves for three cases of knot vector

From the knot vector, basis functions $N_{i,p}$ and rational basis functions $R_{i,p}$ are established (see Eq. (4) and Eq. (5)). The displacement function is computed from calculating the control points in Eq. (8). The results of the displacement, velocity, acceleration, and jerk diagram (SVAJ diagram) show in Fig. 1. It is seen that the difference of the displacement curves is not changed much. However, the velocity, acceleration, and jerk curves are much different. The maximum values of velocity, acceleration, and jerk using chord length method are much smaller than others.

As the second example, we consider the cardiovascular mock loop where the motion of the human heart is simulated. The measurement of the displacement fol-

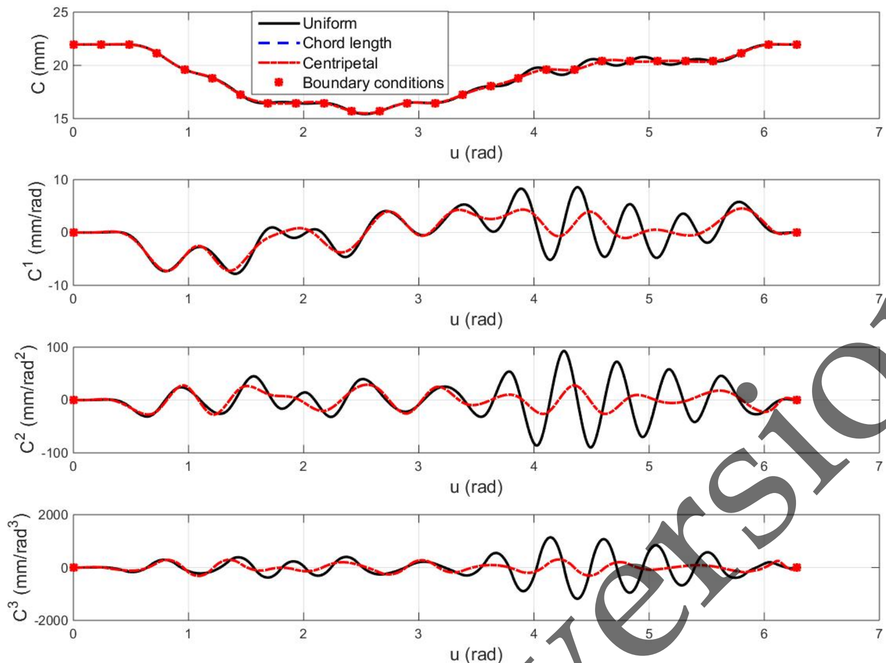

Fig. 2 SVAJ diagram comparison of uniform, chord length, and centripetal methods

lower is shown by star signs in Fig. 2 with 27 values of displacement. Because of the discontinuity of the velocity and acceleration, the infinite values of the acceleration and jerk will occur, respectively. Thus, to avoid the discontinuity of the velocity, acceleration, and jerk, some boundary conditions are added at the start and the end points of the velocity, acceleration, and jerk, such that their values are equal to zero. Respectively, the input angle vector with 33 elements is written as $\mathbf{D} =$ [0, 0, 0, 0, 0.2417, 0.4833, 0.725, 0.9666, 1.2083, 1.45, 1.6916, 1.9333, 2.1749, 2.4166, 2.6583, 2.8999, 3.1416, 3.3833, 3.6249, 3.8666, 4.1082, 4.3499, 4.5916, 4.8332, 5.0749, 5.3165, 5.5582, 5.7999, 6.0415, 6.2832, 6.2832, 6.2832, 6.2832].

Fig. 2 shows SVAJ diagram in one cycle of the cam mechanism. The displacement, velocity, acceleration, and jerk curves in case of the chord length and the centripetal method are coincided because of the same as vector $\mathbf{t}$, also knot vector $\mathbf{U}$. As $\mathbf{D}$ above, the difference between two elements, $|D_i - D_{i-1}|\,(i = 5, \ldots, 30)$, does not change and the remaining elements are equal to zero. In this case, the parameters t_i are not affected by the power α of the centripetal method. Thus, they have similar values in both the chord length method and the centripetal method, likewise the value of knots u_i. As shown in Fig. 2, the displacement for the uniformly spaced method is slightly oscillating. Therefore, the peak values of the velocity, acceleration, and jerk curves occur. The maximum values of velocity, acceleration, and jerk with the chord length and the centripetal methods are much smaller than the uniformly spaced method.

5 Conclusions

Using NURBS curve for cam motion synthesis is flexible and robust because it satisfies arbitrary boundary conditions of displacement, velocity, acceleration, and jerk constraints. Furthermore, NURBS curve and its derivative are controlled by several parameters such as knot vector, control points, and weights. The evaluation of effecting the knot vector on the displacement, velocity, acceleration, and jerk curves is presented in this paper. Several methods for computing the knot vector of NURBS used to synthesize the motion curves are presented. The results show that the maximum values of acceleration and jerk in case of the chord length method are smaller than the other methods. Especially, these values for chord length method are much smaller than the uniform method.

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