Design Optimization and Accuracy Analysis of a Planar 2PRP-PRR Parallel Manipulator

S. Mohan[1], B.Corves[2] and P. Wenger[3]

[1] *Discipline of Mechanical Engineering, Indian Institute of Technology Indore, Indore 453552, India, e-mail: santhakumar@iiti.ac.in*

[2] *Department of Mechanism Theory and Dynamics of Machines, RWTH Aachen University, Aachen 52072, Germany, e-mail: corves@igm.rwth-aachen.de*

[3] *IRCCyN - UMR CNRS 6597, Ecole Centrale de Nantes, Nantes 44321, France, e-mail: Philippe.Wenger@ls2n.fr*

Abstract. This paper addresses the dimensional synthesis and design optimization of a three-degree-of-freedom planar U-shape fixed base 2PRP-PRR parallel manipulator to maximize its workspace. Two kinematic design solutions are proposed and their link parameters are optimized to maximize the workspace. Furthermore accuracy analysis of the optimized manipulator configurations for the actuator inaccuracies is performed and the results are compared with the well-known planar 3PRP and 2PRP-PPR parallel configurations.

Key words: Design optimization, Planar parallel manipulator, 2PRP-PPR, Workspace, Accuracy analysis, Error analysis.

1 Introduction

Planar parallel manipulators are getting great attention and interest for industrial applications namely positioning and tracking in recent years. Although planar parallel manipulators have several advantages such as higher accuracy, speed, rigidity and payload capability, they have shortcomings due to smaller workspace and presence of singularities [1, 2]. Therefore, several researchers are working towards identifying the best possible (optimal) configuration to overcome these shortcomings [2]. In order to identify the optimal configuration, there are several methods applied and quantifiers used in the literature [2-4]. For example, the dexterity or isotropy index, global conditioning index, payload index, accuracy measures, etc. are being used to quantify the performance of the manipulator [2-4]. From the literature, it is found that usage of unsymmetrical fixed base (U-shape fixed base) provides larger singularity-free workspace and simple kinematic relations rather than symmetrical fixed base (Δ-shape fixed base) [4, 7]. Furthermore it is found that planar parallel manipulators having their first joint actuated and prismatic arranged in a U-shape fixed base provide better performance and few advantages over other configurations [4, 7]. The detailed kinematic and dynamic performance analyses of this particular family was performed and it was found that the planar 2PRP-PRR parallel configuration has better performance in terms of isotropy and payload indices [4]. The 2PRP-PRR manipulator kinematics and its kinematic performance measures were presented in [5]. In [5], the optimum kinematic design of the configuration, i.e., the optimal link parameters were not considered, but the performance results confirmed the influence and sensitivity of the link parameters in overall performance. During the initial design procedure, the analysis of the performance sensitivity to uncertainties is an important task and sensitivity analysis of planar parallel manipulators was performed using the screw theory [8]. In [8], the end-effector pose errors due to dimensions and actuators errors were calculated and compared for different configurations.

Therefore, in this paper the design optimization of the planar 2PRP-PRR parallel configuration is performed with two cases namely the PRR leg connections with and without offset distance between active prismatic joint (slider block) to the RR link. The configurations of these cases are given in Figs. 1 and 2. Furthermore the optimal configuration workspaces are compared with well-known planar 3PRP and 2PRP-PPR parallel configurations. In addition, accuracy analyses of the optimal configuration for the actuator inaccuracies are performed and compared through the help of analytical approach [7] based on forward kinematic relations. For the accuracy measure analysis, the local maximum position errors of the end-effector for a given actuator inaccuracies and a common test region within the singularity-free workspace are considered.

The remainder of this paper is arranged as follows: the next section presents the mathematical background which includes kinematic relations of the manipulator whose workspace will be studied in this paper. Section 3 presents the design optimization results obtained from the genetic algorithms and Section 4 presents the accuracy measure in terms of error analysis which computing the local maximum position errors for the given actuator inaccuracies. Conclusions and scope of future work are given in the last section.

2 Mathematical Background

The kinematic arrangements (both cases) of the manipulator are shown in Figs. 1 and 2. The fixed base, 0, and the moving platform (end-effector), 7 are connected through three legs. In these three legs: two of them have prismatic, revolute and prismatic joints and the third leg has prismatic, revolute and revolute joints. In all three legs, the starting prismatic joint is actuated and other joints are passive. The vector of actuator coordinates (joint displacements) is $\mathbf{q} = \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix}^{\mathrm{T}}$ and these joint displacements are considered as positive values, i.e., $r_1 \geq 0, r_2 \geq 0$ and $r_3 \geq 0$.

The vector of task coordinates of the end-effector is $\boldsymbol{\mu} = \begin{bmatrix} x & y & \theta \end{bmatrix}^{\mathrm{T}}$. The forward kinematic relation of the manipulator is as follows:

$$\boldsymbol{\mu} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = \begin{bmatrix} r_1 + l_4 \cos\theta_4 + l_{1'} \cos\theta_1 \\ r_2 + \left(\dfrac{r_3 - r_2}{s}\right)\left(r_1 + l_4 \cos\theta_4 + l_{1'} \cos\theta_1\right) \\ \tan^{-1}\left(\dfrac{r_3 - r_2}{s}\right) \end{bmatrix} \tag{1}$$

where, $\theta_4 = \operatorname{atan2}\left(\dfrac{y - l_{1'} \sin\theta_1}{l_4}, \sqrt{1 - \left(\dfrac{y - l_{1'} \sin\theta_1}{l_4}\right)^2}\right)$, s is the longitudinal span. $l_{1'}$ and l_4 are the link lengths of link 1' and link 4.

The inverse kinematic relations of the manipulator are given as follows:

$$\mathbf{q} = \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} x - l_4 \cos\theta_4 - l_{1'} \cos\theta_1 & y - x\tan\theta & y + [s - x]\tan\theta \end{bmatrix}^{\mathrm{T}} \tag{2}$$

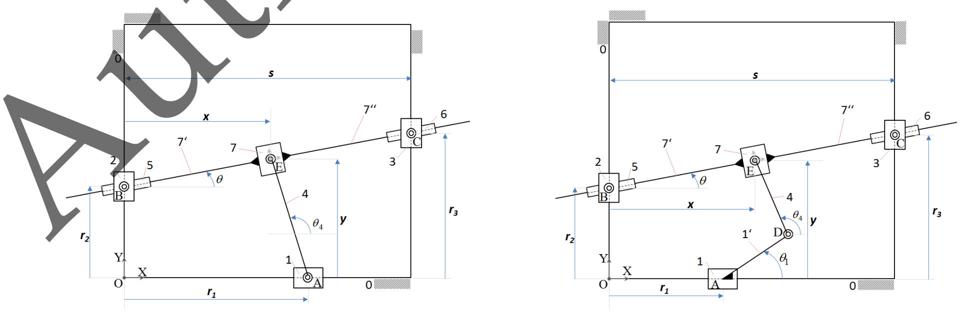
Velocity relations can be obtained by differentiating (2) with respect to time, as

$$\dot{\mathbf{q}} = \mathbf{J}(\mu)\dot{\boldsymbol{\mu}} \tag{3}$$

where, $\mathbf{J}(\mu)$ is the Jacobian matrix of the kinematic configuration and given as:

$$\mathbf{J}(\mu) = \begin{bmatrix} 1 & \tan\theta_4 & 0 \\ -\tan\theta & 1 & -x\sec^2\theta \\ -\tan\theta & 1 & (s - x)\sec^2\theta \end{bmatrix} \tag{4}$$

where, $\mathbf{J}(\mu)$ is singular at $(1 - \tan\theta \tan\theta_4) = 0$, in other words, singularity is encountered whenever rod 4 is perpendicular to rods 7' and 7". In this case the moving platform can perform infinitesimal translation motions along the direction of 7' and 7". Therefore singularity-free workspace computation is performed and described in the next section.



(a) Case 1: without an offset distance **(b)** Case 2: with an offset distance

Fig. 1 Schematic arrangements of the planar 2PRP-PRR parallel manipulator

3 Design Optimization and Workspace Analysis

Manipulator design is one of the complex subjects and the overall performance heavily depends on the manipulator geometry and the performance quantifiers are almost depend on the geometry as well [2, 6]. Therefore, design optimization is an essential process in manipulator design and this is an iterative process. In this paper, the manipulator's geometry is optimized by maximizing the singularity free workspace. One of the design parameters can be fixed without loss of generality. Here the longitudinal span (s) is assumed to be 0.2 m. The design optimization of the configuration 1 (without offset distance) is performed with the help of a simple scanning method. In this method, the link length (L_{AE}) is varied from 0.05 m to 0.2 m and the area of the singularity-free workspace is calculated for the constant end-effector orientation in [-15°, +15°]. That is, the set of points reachable for any orientation within [-15°, +15°]. The area of the singularity-free workspace as function of the link length is plotted in Fig. 3. It shows that the largest area is obtained for L_{AE} = 0.148 m and its numerical value is 0.0136 m^2. This value still decreases when increasing the value of L_{AE} above 0.2 m. However, it is limited to the longitudinal span of the manipulator.

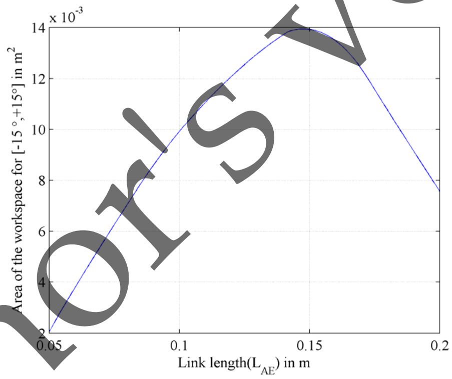


Fig. 3 Area of the workspace of the 2PRP-PRR (case 1) as function of the link length variations

Design optimization of configuration 2 is not as simple as the earlier. This configuration has three design variables namely, θ_1, L_{AD} and L_{DE}. The influence of each parameter's variations on the workspace is presented in Fig. 4. It shows that each design variable has a significant contribution. Therefore, in this work the design optimization is performed with the help of genetic algorithms for maximizing the workspace. The area of the singularity-free workspace is calculated for the given entire stroke length of each joints (it is considered as 0 to 0.2 m for all joints) and finding the points which give non-zero determinant value of the Jacobian matrix. Here for the optimization, the end-effector positions, x and y are varied from -0.1 m to 0.3 m and the end-effector orientation θ is varied from -15° to +15°. The genetic algorithm optimization toolbox in matlab is used for the numerical computation. The optimized values of design variables for the constant end-effector orientation in [-15°, +15°] are obtained as follows: $\theta_I = 47.15°$, L_{AD} = 0.1376 m and L_{DE} = 0.1045 m. The area of singularity-free workspace is 0.0239 m^2. The constant workspace optimization process is also performed for different orientations of the end-effector. The variations in the optimized design values are very minimal. These configurations based on their optimized design values are compared with well-known planar 3PRP and 2PRP-PPR manipulators and, their kinematic arrangements are presented in Figs. 5 and 6. For better comparison, the span (s) is considered as 0.2 m for all manipulators. The singularity-free constant orientation workspaces of these configurations are presented in Fig. 7. Areas of the singularity-free workspace of the manipulators are given in Table 1.

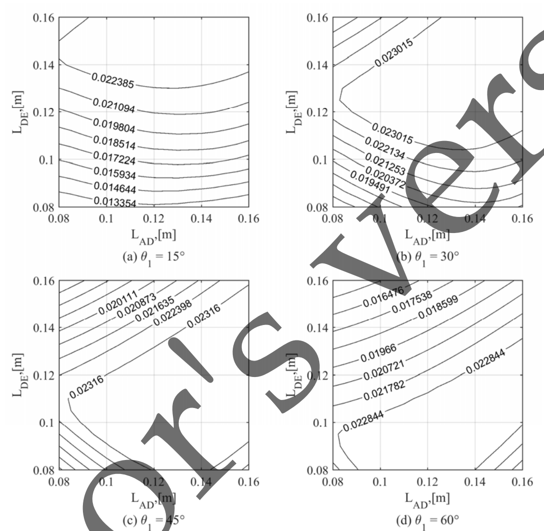


Fig. 4 Contour plots of the workspace area of the 2PRP-PRR (case 2) configuration for the parameter variations

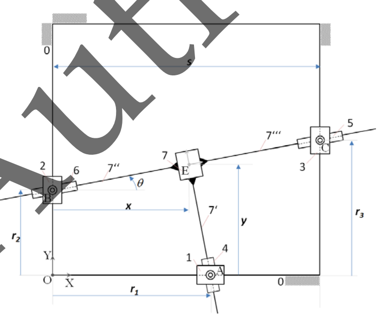


Fig. 5 Schematic arrangement of the planar 3PRP parallel manipulator

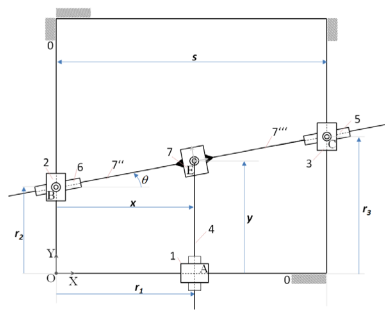


Fig. 6 Schematic arrangement of the planar 2PRP-PPR parallel manipulator

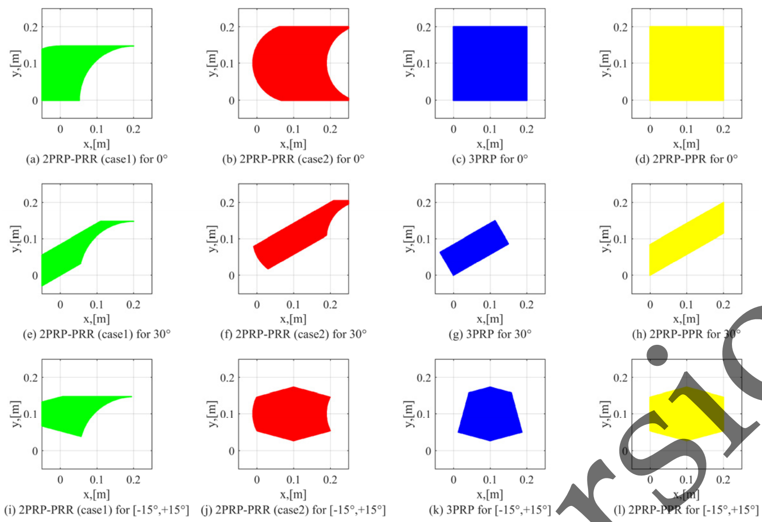


Fig. 7 Singularity-free workspace plots of planar parallel manipulators

Table 1. Singularity-free workspace of the planar parallel manipulators

Configuration	***Singularity-free workspace in*** **m^2**		
	θ = 0°	**θ = + 30°**	**θ = [-15°,+15°]**
3PRP	0.0400	0.0127	0.0185
2PRP-PPR	0.0400	0.0169	0.0240
2PRP-PRR (case 1)	0.0196	0.0122	0.0137
2PRP-PRR (case 2)	0.0400	0.0175	0.0239

4 Error Analysis

Most of the kinematic performance measures depend on the Jacobian matrix which may not be appropriate for overall performance comparison and they are generally not appropriate to manipulators with mixed units. Therefore, accuracy measure based on kinematic error analysis is performed. In this error analysis, the end-effector position errors due to actuator inaccuracies (index errors) are only considered [7]. Since the end-effector position error of the 2PRP-PPR is constant in its entire workspace for a given end-effector orientation [7], the error analysis is carried out only for 3PRP and 2PRP-PPR (case 2) configurations. The test regions are chosen within the singularity-free workspace and presented in Fig. 8. The limit of actuator inaccuracies (maximum) of all active joints are considered to be equal to ± 50 μm. For obtaining the local maximum end-effector pose errors based on the above range of error parameters is considered as a maximization (optimization) problem. In this paper, the maximization of local position and orientation errors of the end-effector is carried out using one of the popular optimization methods namely genetic algorithms. In this method, the local position errors are maximized based on the forward kinematic model, actuator inaccuracies range.To find the maximum value of the pose error, the in-build MATLAB function namely ‘***ga***’ is used. The error contours of these manipulators for different end-effector orientations are presented in Fig. 9. The maximum, minimum and mean values of local maximum end-effector position errors of the manipulators are given in Table 2.

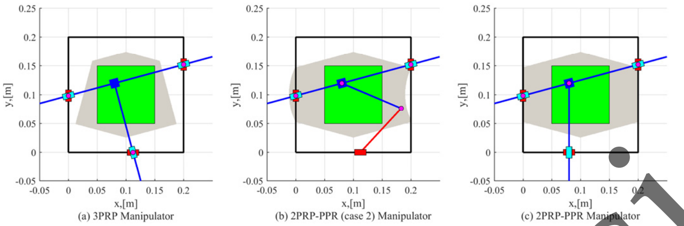


Fig. 8 Singularity-free workspace for [-15°, +15°] along with the accuracy test region

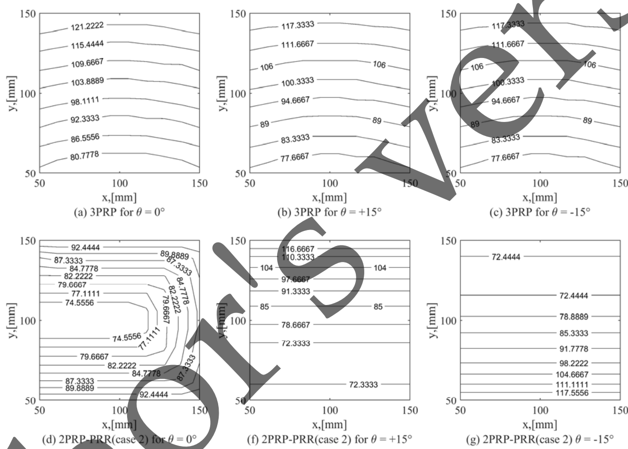


Fig. 9 Contour plots of the maximum position error (in µm)

Table 2. Local maximum end-effector position errors of the planar parallel manipulators

Configuration		***Local maximum end-effector position errors (in µm)***				
		θ = 0°	**θ = +15°**	**θ = + 30°**	**θ = -15°**	**θ = -30°**
3PRP	Maximum	127.48	123.15	110.42	123.14	110.42
	Minimum	75.05	72.47	65.01	72.45	64.98
	Mean	101.28	97.83	87.72	97.83	87.72
2PRP-PRR (case 2)	Maximum	95.74	123.74	177.13	124.51	179.04
	Minimum	68.73	66.31	59.33	66.15	61.63
	Mean	84.62	87.43	102.28	87.65	102.95
2PRP-PPR	Constant	70.71	80.74	93.38	80.74	93.38

From the results, it is found that 2PRP-PRR configuration is better in terms of accuracy in all three cases as compared to 3PRP configuration. Not only smaller values but also the range of smaller values is much larger as well. Further, error values of 2PRP-PRR depend on the location of the end-effector in the workspace.

Further, an experimental study on energy consumption with payload of these three manipulators was carried out in [4]. The values of energy consumption for a circular-path tracking task with full payload (50 N) of the 3PRP, 2PRP-PPR and 2PRP-PRR (case 2) are 0.846 Wh, 1.349 Wh and 0.739 Wh [4]. The 2PRP-PPR configuration is better in terms of accuracy but the PPR leg has a moving passive prismatic joint which requires more energy and driving force compare to PRR leg [4]. Therefore, in overall, it is found that the optimized 2PRP-PRR configuration with the offset distance is could be better as compared to other configurations.

5 Conclusions

In the present paper, the design optimization of the planar 2PRP-PRR parallel manipulator was performed. Two different configurations were considered and their parameters were optimized. The constant orientation workspaces of these configurations were found and compared. Based on workspace results, it was found that configuration 2 (with offset distance) has better performance than the configuration 1 (without offset distance). Further error analysis was performed for the configuration 2 and compared with well-known 3PRP and 2PRP-PPR configurations. From the overall results, the optimized 2PRP-PRR design could be a better planar parallel platform for precise and accurate positioning applications. The use of the proposed optimum 2PRP-PRR manipulator in conjunction with the mechanical error compensation motion control would be the next research objective.

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