A Novel Kinematic Model of Spatial Four-bar Linkage RSPS for Testing Accuracy of Actual R-Pairs with Ball-bar

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Abstract. A novel kinematic model of spatial four-bar linkage RSPS is firstly presented for accuracy testing of R-pairs. During accuracy testing, the kinematic chain SPS of the ball bar and the R-pair constitute a RSPS mechanism, while the structure parameters of the RSPS mechanism correspond to the mounting parameters of the ball bar. Thus, the mounting position errors of the ball bar are identified by using the kinematic synthesis of the RSPS mechanism, based on the discrete data measured by the ball bar. Furthermore, the relationships between the measured errors and the mounting parameters of the ball bar are analyzed, by using the solutions of the kinematic equations of the RSPS mechanism.

Key words: spatial four-bar linkage; kinematics; ball bar; error identification; accuracy

1 Introduction

The ball bar is a widely used instrument for accuracy testing of machine tools. It was invented by Bryan, and firstly used to measure the errors of the simultaneous motion of two prismatic pairs [1, 2]. Then, a lot of ball bar methods are presented to measure the geometrical errors of the machine tools and robots [3-5]. These methods improved the efficiency of error testing and calibration, especially for multi-axis motions.

For a rotary pair, or R-pair, the accuracy is often represented by radial and axial runouts of the rotor [6, 7]. However, the runouts are influenced by the geometric errors of the measured surface, and related with the mounting positions of the sensors and work-pieces. Some precise artifacts are used as the work-pieces to reduce the influences of geometric errors; these methods are high-precision but time consuming, because it is difficult to mount the work-pieces to a suitable place.

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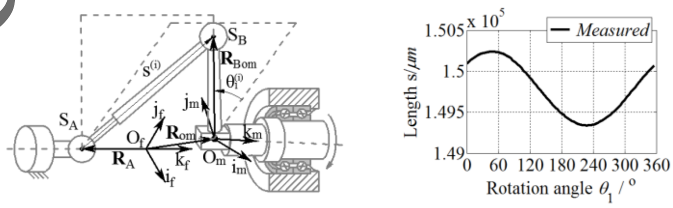
As the ball bar is easy installation and efficient for motion measurement, some ball bar methods are presented to measure the accuracy of R-pairs [8-11]. In these researches, the mounting position errors of the ball bar are eliminated by adjusting devices, or separated from the measured results by least square circle fitting. The former is time-consuming and the later is based on the assumption of planar motion. In fact, the motions of the ball bar are spatial motions during accuracy testing, and measured results are related with the mounting position of the ball bar. Therefore, how to identify the mounting position errors from the measured data? What are the relationships between the measured data and the mounting positions? These questions are discussed in this paper. A novel method, based on the spatial four-bar linkage RSPS, is firstly presented to identify the mounting errors of the ball bar, and the relationships between the measured errors and the mounting parameters are analyzed by using the geometrical model of the RSPS mechanism.

2 Accuracy Testing of R-Pairs with Ball Bar

A ball bar is composed of two master balls and a precise linear displacement sensor, as shown in Fig. 1. Thus, the result measured by the ball bar is the distance between the centers of the two balls.

Fig. 1 A ball bar with two master balls and one displacement sensor

During accuracy testing of an R-pair, the fixed ball S_A is mounted on a base and the moving ball S_B is mounted on the rotor, as shown in Fig. 2a. In most cases, the displacement sensor is possibly located at the sensitive direction of the rotary errors, such as the axial and radial directions [10]. As the rotor rotates, the ball bar records the distances $s^{(i)}$ between the centers of the two balls at every rotary position. For example, the measured results for one round are shown in Fig. 2b.



(a) (b)

Fig. 2 Accuracy testing of an R-pair with ball bar and the measured data

Generally, the ball S_A is expected to be mounted on the rotational axis of the R-pair, and the measured results are desired to be pure error motions caused by the R-pair. Unfortunately, the mounting position errors of two balls are inevitable. Sometimes, the mounting position errors are much bigger than the rotary errors of the R-pair, which make the results measured distribute as a trigonometric function, as shown in Fig. 2b. Thus, it is necessary to eliminate the influences of the mounting position errors.

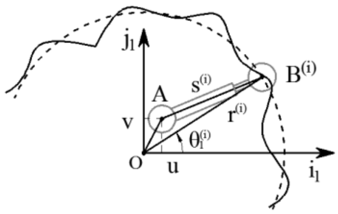
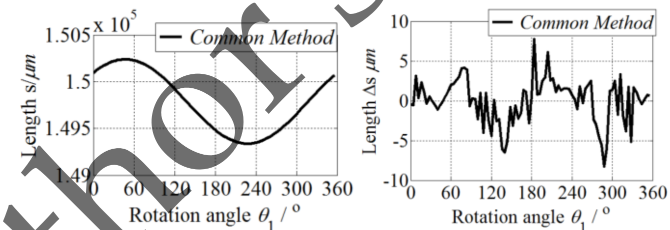


Fig. 3 The least square circle fitting of the measured results

In the existing researches [10], the mounting errors, or the eccentric motions, are often separated by using the least square circle fitting. As shown in Fig. 3, the motion of the ball bar is assumed as a planar motion, and the mounting errors of the ball bar are equivalent to the eccentricity (u, v) of the center point A relative to the rotary axis. Then, the eccentric motions caused by the mounting errors and the rotary error motions of the center point B will be separated. The results are shown in Fig. 4.



(a) The eccentric motions (b) The rotary error motions

Fig. 4 The eccentric motions and rotary error motions separated by least suquare fitting

3 Mechanism Model of Ball Bar for Testing R-Pair

The kinematic chain SPS of the ball bar and the R-pair constitute a spatial RSPS mechanism, while the structure parameters of the RSPS mechanism correspond to the mounting parameters of the ball bar. The output motions of the prismatic pair correspond to the motions caused by the mounting positions errors of the ball bar, if the rotary pair rotates without errors. Thus, the mounting positions errors can be identified by kinematic synthesis of the RSPS mechanism.

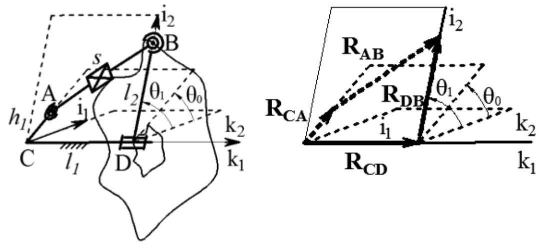


Fig. 5 The spatial RSPS mechanism and its parameters

In order to analyze the motions of the mechanism, a fixed frame {C; $\mathbf{i}_1$, $\mathbf{j}_1$, $\mathbf{k}_1$} is employed and fixed to the base of the R-pair. Meanwhile, a moving frame {D; $\mathbf{i}_2$, $\mathbf{j}_2$, $\mathbf{k}_2$} is employed and attached to the moving ball S_B. In the fixed frame, the displacement equation of the RSPS mechanism can be written as

$$\mathbf{R}_{AB} = \mathbf{R}_{CD} + \mathbf{R}_{DB} - \mathbf{R}_{CA} \tag{1}$$

where, $\mathbf{R}_{AB}$ denotes the vector from the center points A to B, whose length $|\mathbf{R}_{AB}|=s$ is the distance measured by the ball bar. The equation (1) can be rewritten as

$$s = \sqrt{l_1^2 + l_2^2 + h_1^2 - 2l_2h_1\cos(\theta_1 - \theta_0)} \tag{2}$$

where, l_1, l_2, h_1 and θ_0 are the structure parameters of the RSPS mechanism, as shown in Fig. 5. The parameters h_1, θ_0 and l_1 locate the position of the center A, and the parameter l_2 determines that of the center B. θ_1 is the rotary angle.

Currently, the distances measured by the ball bar, denoted by $s^{(i)*}$ (i=1,2,…,n), contain both the motion caused by the mounting errors and the error motion of the R-pair. In order to identify the mounting position errors, a mathematic model of kinematic synthesis of an RSPS mechanism is set up, in which the mounting parameters are equivalent to the structure parameters of RSPS, that is

$$\begin{cases} \Delta_s = \min\limits_{\mathbf{x}} \max\limits_{1\le i\le n} \left\{\Delta^{(i)}(\mathbf{x})\right\} = \min\limits_{\mathbf{x}} \max\limits_{1\le i\le n} \left\{\left|s^{(i)*} - s^{(i)}\right|\right\} \\ s.t.\quad l_1, l_2, h_1 \in (-\infty, +\infty);\ \ \theta_0 \in (0, 2\pi) \\ \mathbf{x} = (l_1, l_2, h_1, \theta_0)^T \end{cases} \tag{3}$$

where, Δ_s is the maximum deviation between the distances measured by the ball bar and those calculated by the RSPS mechanism. n is the number of the discrete rotary positions. The output motions $s^{(i)}$ at the rotary position i is

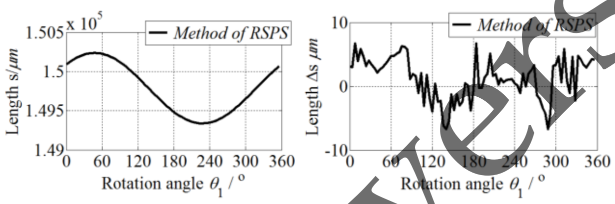
$$s^{(i)} = \sqrt{l_1^2 + l_2^2 + h_1^2 - 2l_2h_1\ \cos(\theta_1^{(i)} - \theta_0)} \tag{4}$$

Based on the measured data, a four-bar linkage RSPS with parameters (l_0, l_1, h_0 ,θ_0) can be optimally calculated by the equation (3), shown in Table 1.

Table 1. The parameters of RSPR corresponding to the ball-bar testing

l_1 / *um*	l_0 / *um*	h_0 / *um*	θ_0 /*rad*	Δ_s / *um*
63198.6	135798.9	1063.8	2.3114	6.7

The output displacements $s^{(i)}$ of the RSPS mechanism can be calculated by equation (4) with the identified parameters, as shown in Fig. 6a; and the remained rotary errors of the center point B in the measuring direction are shown in Fig. 6b.



(a) (b)

Fig. 6 The output displacements of RSPS mechanism and the rotary errors

4 RSPS Mechanism and Mounting Errors

In order to discuss the relationships between the mounting parameters of the ball bar and the measured results, the geometrical model of the RSPS mechanism is set up, as shown in Fig. 7. The center-line AB of the ball bar traces a conical surface Σ_{AB} in the fixed frame [11]; the parameters are the same as the RSPS mechanism.

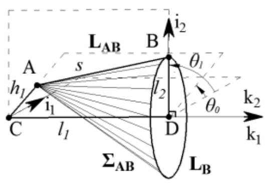


Fig. 7 The geometrical model the RSPS mechanism

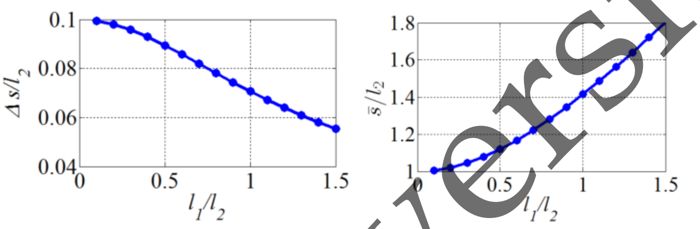
For convenience, the equation (2) is divided by l_2 to avoid the influences caused by the rotary radius, that is

$$\frac{s}{l_2}=\sqrt{\left(\frac{l_1}{l_2}\right)^2+\left(\frac{h_1}{l_2}\right)^2+1-2\frac{h_1}{l_2}\cos(\theta_1-\theta_0)} \tag{5}$$

Based on equation (5), the relationships between the mounting parameters and the measured results are discussed as follow.

(1) s/l_2 and l_1/l_2

The parameter l_1/l_2 is inversely proportional to the cone angle of the conical surface, shown in Fig.7. s/l_2 and l_1/l_2 represent the influences of the mounting direction of the ball bar. The curves in Fig. 8 show that the amplitude of s/l_2 is proportional to the cone angle and the mean value of s/l_2 is inversely proportional to the cone angle.



(a) Amplitude of s/l_2 (b) Mean value of s/l_2

Fig. 8 The relationships between s/l_2 and l_1/l_2

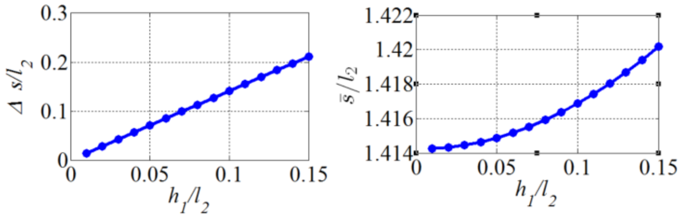
In particularly, if the parameter l_1/l_2 is zero, the spatial RSPS mechanism degenerated to be a planar mechanism. In this case, the axial mounting position errors of center points A and B are eliminated; the results measured by the ball bar are radial run-out of the R-pair. The equation (2) can be written as

$$s=\sqrt{l_2^2+h_1^2-2l_2h_1\cos(\theta_1-\theta_0)} \tag{6}$$

There are three parameters h_1, l_2 and θ_0 in equation (6), corresponding to the radial mounting errors of center points A and B, which are the same as the least square circle fitting discussed in section 2.

(2) s/l_2 and h_1/l_2

The parameter h_1/l_2 is proportional to the eccentricity of the conic node A, shown in Fig. 7. The relationships between s/l_2 and h_1/l_2 represent the influences of the mounting position errors of the ball S_A. The calculated results, shown in Fig. 9, reveal both of the amplitude and mean value of s/l_2 are proportional to the eccentricity of the conic node.



(a) Amplitude of s/l_2 (b) Mean value of s/l_2

Fig. 9 The relationships between s/l_2 and h_1/l_2

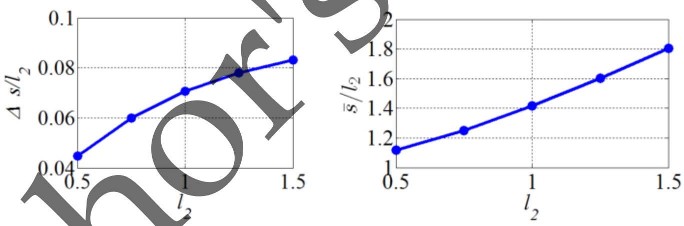
In particularly, if the parameter h_1/l_2 is zero, the radial mounting position error of the center point A is eliminated. The equation (2) can be written as

$$s = \sqrt{l_1^2 + l_2^2} \tag{7}$$

Two parameters l_1 and l_2 in equation (7) correspond to the position of the center point A and the rotary radius of the center points B.

(3) s/l_2 and l_2

The parameter l_2 corresponds to the rotary radius of the center point B, thus, the relationships between s/l_2 and l_2 represent the influences of the mounting position of the moving ball S_B. The calculated results, shown in Fig. 10, express that the amplitude and mean value of s/l_2 is proportional to the rotary radius.



(a) Amplitude of s/l_2 (b) Mean value of s/l_2

Fig. 10 The relationships between s/l_2 and l_2

(4) s/l_2 and θ_0

As known in equation (2), the amplitude and mean value of s/l_2 is independent of the parameter θ_0, but the phase of s/l_2 is. In particularly, if the parameter θ_0 is zero, the equation (2) can be written as

$$s = \sqrt{l_1^2 + l_2^2 + h_1^2 - 2l_2 h_1 \cos\theta_1} \tag{8}$$

Three parameters l_1, l_2 and h_1 in equation (8) correspond to the axial and radial errors of the center points A and B.

5 Conclusions

(1) The kinematic model of an RSPS mechanism is presented for accuracy testing of R-pair with ball bar. The structure parameters of the RSPS mechanism correspond to the mounting parameters of the ball bar, and the mounting errors can be identified by kinematic synthesis of an RSPS mechanism.

(2)The amplitude of s/l_2 is proportional to the cone angle, and the mean value of s/l_2 is inversely proportional to the mounting angle of the ball bar.Both of the amplitude and mean value of s/l_2 are proportional to the eccentricity of the balls S_A and S_B.

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