Robust Optimization of the RAF parallel robot for a prescribed workspace

M. A. Laribi[1], A. Mlika[2], L. Romdhane[2,3] and S. Zeghloul[1]

[1] Dept. of GMSC, Pprime Institute, CNRS - University of Poitiers - ENSMA - UPR 3346, France, e-mail: med.amine.laribi@univ-poitiers.fr said.zegloul@univ-poitiers.fr

[2] Mechanical Laboratory of Sousse (LMS), National Engineering School of Sousse, University of Sousse, Sousse 4000, Tunisia, e-mail: abdelfattah.mlika@gmail.com lotfi.romdhane@gmail.com

[3]Mechanical Engineering Department, American University of Sharjah, PO Box 26666, Sharjah, United Arab Emirates, e-mail: lromdhane@aus.edu

Abstract. This paper deals with the optimal synthesis of the RAF robot for a prescribed workspace. The RAF (Romdhane-Affi-Fayet) robot is a three translational parallel manipulator (3TPM). A method based on the genetic algorithm is used to solve the optimization problem. A multi-objective function, based on the mathematical concept of the power of a point with respect to a surface, is formulated. The suggested method is simple and effective in defining the geometry of the robot having the smallest workspace that includes a specified volume and the best kinematic performance.

Key words: optimal design, synthesis, RAF parallel robot, genetic algorithm, workspace, power of a point, dexterity index.

1 Introduction

The interest in parallel manipulators (PM) arises from the fact that they exhibit high stiffness in nearly all configurations and a high dynamic performance. The RAF (Romdhane-Affi-Fayet) parallel manipulator is also a 3TPM and it consists of a mobile platform connected to the base by three active legs and two passive kinematics' chains [1, 2, 3].
The design problem has been addressed in many previous works [6,11,12,13,14 16,17,18,19]. In [9], we showed using the mathematical concept of the power of a point, how to design a DELTA robot for a prescribed workspace. In this paper, we will solve the problem of designing the three translational dof RAF robot to have a specified workspace and the highest dexterity. A multi-objective genetic algorithm

(MOGA) is used to solve the optimization problem, because of its robustness and simplicity.
This paper is organized as follows: Section 2 presents the architecture of the RAF robot. Section 3, is devoted to the kinematic analysis and the determination of the workspace of the RAF parallel robot. The dexterity index of the robot is presented in Section 3. In Section 4, we carry out the formulation of the optimization problem using the genetic algorithm. Section 5 contains the results and discussion. Finally, Section 6 contains some conclusions.

2 Architecture of the RAF parallel robot

The RAF robot consists of a mobile platform connected to the base by 3 legs. These three legs constitute the actuators of the manipulator, whereas two other kinematic chains with passive joints are used to eliminate the three rotations of the mobile platform with respect to the base (Fig. 1) [1, 2].
Let R_b (O_B, x_B, y_B, z_B) and $R_P(P, x_P, y_P, z_P)$ represent two references frames, which are fixed on the base and on the platform, respectively (see Fig. 1). The active legs are connected to the base through spherical joints. These spherical joints are centered in points B_i, $i = 1,2,3$, with the base and in points C_i, $i = 1,2,3$, with the platform.

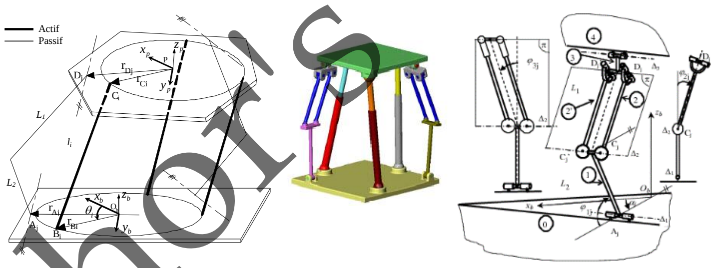

Fig. 1 The RAF robot parameters.

In this work, a standard configuration is selected for the active legs as follows :

• $r_{Bi} = r_B$; $r_{Ci} = r_C$ $(i = 1, 2, 3)$ which means that the centers of the spherical joints relating the three legs to the base, respectively the platform, are located on a circle centered in O_B, respectively P, and with a radius r_B, respectively r_C.

• $\theta_{C1} = \theta_{B1} = \theta_1 = 0$; $\theta_{C2} = \theta_{B2} = \theta_2 = \frac{2\pi}{3}$; $\theta_{C3} = \theta_{B3} = \theta_3 = \frac{4\pi}{3}$ which means that the three centers are arranged at $120°$ from each other.

The parameters of the active kinematics' chains are:

• $l_{\max}$: The maximum extension of the active legs.

• $l_{\min}$: The minimum extension of the active legs.

Fig. 1 shows the architecture of one of the passive kinematics chains [2]. Each kinematic chain is made of an arm (1) connected to the base (0) by a revolute joint. More details on the RAF architectrure are presented and discussed in [1, 2, 3].

The parameters of the passive kinematics' chains are L_1 and L_2 (see Fig. 1). We will take the case where $L_1 = L_2 = L$. Points A_j, respectively D_j ($j = 1{,}2$), are located on a circle with a radius r_A, respectively r_D. We also have $(A_1\widehat{O_B}A_2) = 120°$ (see Fig. 1).

3. Workspace of the RAF robot

The workspace of the RAF robot is the intersection of two workspaces of the two imbricated robots, respectively, the passive part and the active part.

3.1 Active and passive workspaces of the platform

The active workspace of the RAF robot is defined by a volume, in the Cartesian space, reachable by the center of the platform $P[X_P, Y_P, Z_P]$. The geometrical model of the active kinematic chain is described by the following equation for each actuator (for $i = 1, .., 3$):

$$(R\cos\theta_i - X_P)^2 + (R\sin\theta_i - Y_P)^2 - Z_P^2 - l_i^2 = 0 \qquad (1)$$

It is assumed that the actuators are identical and their lengths vary between the minimal value , l_{min}, and the maximum value, l_{max} ($l_{min} = l_{max}/3$). The reachable points of each one of these legs are confined within a volume delimited by two concentric spheres given by (for $i = 1, .., 3$):

$$(R\cos\theta_i - X_P)^2 + (R\sin\theta_i - Y_P)^2 - Z_P^2 - l_{\max}^2 = 0 \qquad (2)$$

$$(R\cos\theta_i - X_P)^2 + (R\sin\theta_i - Y_P)^2 - Z_P^2 - l_{\min}^2 = 0 \qquad (3)$$

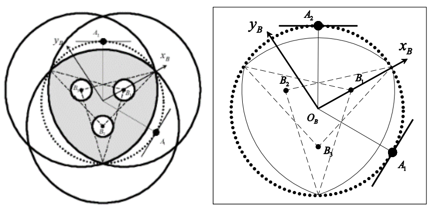

Fig. 2 Slice of the active workspace at XY plane

The intersection of the three volumes delimited by the three pairs of concentric spheres, represents the active workspace of the manipulator for a given orientation.

A slice of the active workspace at $z = z_k$ is shown on **Fig. 2.** This space is similar to that presented by [10] in the case of a Stewart platform of the 6-SPS type. However, our problem is less complex, since we have only three actuators instead of six.

Considering the same point $P[X_P, Y_P, Z_P]$ on the platfom. The kinematic model for the passive chains can be written as, with $j = 1,2$:

$$\mathbf{O}_B\mathbf{P} = \mathbf{O_B A}_j + \mathbf{A}_j\mathbf{C}_j + \mathbf{C}_j\mathbf{D}_j + \mathbf{D}_j\mathbf{P} \tag{4}$$

$$\mathbf{O}_B\mathbf{P} = \begin{bmatrix} r\cos\alpha_j \\ r\sin\alpha_j \\ 0 \end{bmatrix} + \begin{bmatrix} L_2\cos\varphi_{1j}\cos\alpha_j \\ L_2\cos\varphi_{1j}\sin\alpha_j \\ -L_2\sin\varphi_{1j} \end{bmatrix} + \begin{bmatrix} L_1\cos\varphi_{3j}\cos(\varphi_{1j}+\varphi_{2j})\cos\alpha_j \\ L_1\cos\varphi_{3j}\cos(\varphi_{1j}+\varphi_{2j})\sin\alpha_j \\ L_1\cos\varphi_{3j}\sin(\varphi_{1j}+\varphi_{2j}) \end{bmatrix} + \begin{bmatrix} -L_1\sin\alpha_j\sin\varphi_{3j} \\ L_1\cos\alpha_j\sin\varphi_{3j} \\ 0 \end{bmatrix}$$

where, φ_{3j} is the angle between the direction of the 2 forearms and the plane generated by the direction of z-axis and that of the arm, φ_{2j} is the angle between the projection of the forearms on the previously defined plane and the direction of the arm, and φ_{1j} is the angle between the direction of the arm and that of the straight line through O and A_j. In order to eliminate the passive joint variable, we square and add these equations

$$\begin{gathered}[(r + L_2\cos\varphi_{1j})\cos\alpha_j - X_P]^2 + [(r + L_2\cos\varphi_{1j})\sin\alpha_j - Y_P]^2 \\ +[L_2\sin\varphi_{1j} - Z_P]^2 - L_1^2 = 0\end{gathered} \tag{5}$$

where, $r = r_A - r_D$. Equation (5) can be expressed as a function of $\cos\varphi_{1j}$ and $\sin\varphi_{1j}$, as follows:

$$\begin{gathered}(2rL_2 - 2L_2X_P\cos\alpha_j - 2L_2Y_P\sin\alpha_j)\cos\varphi_{1j} - 2rX_P\cos\alpha_j + 2L_2Z_P\sin\varphi_{1j} - 2rY_P\sin\alpha_j \\ +X_P^2 + r^2 + L^2 + Z_P^2 + Y_P^2 - L_1^2 = 0\end{gathered} \tag{6}$$

which can be written as:

$$l_j\cos\varphi_{1j} + m_j\sin\varphi_{1j} - n_j = 0 \tag{7}$$

where,

$u_j = 2rL_2 - 2L_2X_P\cos\alpha_j - 2L_2Y_P\sin\alpha_j$; $m_j = 2L_2Z_P$; $n_j = -2rY_P\sin\alpha_j + X_P^2 + r^2 + L_2^2 + Z_P^2 + Y_P^2 - L_1 - 2rX_P\cos\alpha_j$

Equation (7) can have a solution if and only if for $j = 1,2$:

$$\left|\frac{n_j}{\sqrt{u_j^2+m_j^2}}\right| \le 1 \Leftrightarrow \quad n_j^2 - \left(u_j^2 + m_j^2\right) \le 0 \tag{8}$$

3.2 Workspace of the RAF robot

The workspace of the RAF parallel manipulator is defined by the intersection of the active workspace and the passive one. (see **Fig. 3**).

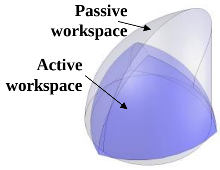

Fig. 3 Workspace of the RAF robot

• Point P is inside the active workspace then, for $i = 1,2,3$:

$$ha_i^{\max}(P) = (R\cos\theta_i - X_P)^2 + (R\sin\theta_i - Y_P)^2 - Z_P^2 - L_{max}^2 \leq 0 \quad (9)$$

$$ha_i^{\min}(P) = (R\cos\theta_i - X_P)^2 + (R\sin\theta_i - Y_P)^2 - Z_P^2 - L_{min}^2 \geq 0 \quad (10)$$

• Point P is inside the passive workspace then, for $j = 1,2$:

$$\begin{aligned} hp_j(P) &= (X_P\cos\alpha_j + Y_P\sin\alpha_j - r)^2 \\ &+(X_P\cos\alpha_j + Y_P\sin\alpha_j)^2 + Z_P^2 + L_2^2 - L_1^2)^2 \\ &-4L_2^2((X_P\cos\alpha_j + Y_P\sin\alpha_j - r)^2 + Z_P^2) \leq 0 \end{aligned} \quad (11)$$

4. Singularity analysis of The RAF robot

Due to the complexity of the kinematic model of parallel mechanisms, most of the authors proposed numerical methods to analyze their singularities. The approach proposed by Romdhane et *al.* [2] to analyze the singularity of the 3-translational-DOF parallel manipulator, is a combination of vector analysis and geometric analysis. Romdhane shows that this method allows to elucidate and physically explain the different singular configurations. The platform can only translate due to the two passive chains even in the absence of the active legs. The architecture of the passive chains is made such that the axis of the revolute joint with the platform is always parallel to the axis of the revolute joint with the base, i.e., the line maintains a constant orientation. The velocity of any point of the platform is the same, i.e.,

$$\mathbf{V}(C_1 \in \wp/B) = \mathbf{V}(C_2 \in \wp/B) = \mathbf{V}(C_3 \in \wp/B) = \mathbf{V}(M \in \wp/B) \quad (12)$$

We can also write that :

$$\dot{l}_i = \frac{V(M\in\wp/B)\cdot B_iP_i^{\mathrm{T}}}{\|B_iP_i\|} = \mathbf{u}_i^{\mathrm{T}} \cdot \mathbf{V}(M \in \wp/B) \tag{13}$$

where $\mathbf{u}_i$ is a unit vector along the leg i and $\dot{l}_i$ is the velocity of the linear actuator located between C_i and B_i. Using matrix representation, we obtain:

$$\begin{bmatrix} \dot{l}_1 \\ \dot{l}_2 \\ \dot{l}_3 \end{bmatrix} = \left[\mathbf{u}_1^{\mathrm{T}}\mathbf{u}_2^{\mathrm{T}}\mathbf{u}_3^{\mathrm{T}}\right][\mathbf{V}(M \in \wp/B)] = \mathbf{J}^{\mathbf{T}}[\mathbf{V}(M \in \wp/B)] \tag{14}$$

where $\mathbf{J}$ is a jacobian matrix whose columns are the unit vectors $(\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3)$. We have the following relation

$$\mathbf{B}_i\mathbf{C}_i = l_i\mathbf{u}_i \tag{15}$$

Where for $i = 1,2,3$,

$$\mathbf{B}_i\mathbf{C}_i = \mathbf{B}_i\mathbf{O}_B + \mathbf{O}_B\mathbf{P} + \mathbf{P}\mathbf{C}_i = \begin{bmatrix} x_P - R\cos\alpha_i \\ y_P - R\sin\alpha_i \\ z_P \end{bmatrix} \tag{16}$$

with $\mathbf{O}_B\mathbf{B}_i = \begin{bmatrix} r_B\cos\alpha_i \\ r_B\sin\alpha_i \\ 0 \end{bmatrix}$, $\mathbf{O}_B\mathbf{P} = \begin{bmatrix} x_P \\ y_P \\ z_P \end{bmatrix}$, $\mathbf{P}\mathbf{C}_{\mathrm{i}} = \begin{bmatrix} x_P + r_C\cos\alpha_i \\ y_P + r_C\sin\alpha_i \\ 0 \end{bmatrix}$

Using Eq. 17 the unit vector $\mathbf{u}_i$ can be expressed as follows :

$$\mathbf{u}_i = \frac{B_iC_i}{\|B_iC_i\|} \tag{17}$$

To evaluate the kinematic performances of robot, researchers have introduced several criteria. The dexterity is a measure reflecting the amplification of error due to the kinematic and statistic transformations between the joints and the Cartesian space. It is of utmost importance that the proposed robot maintains a certain level of dexterity over its workspace. Several criteria were proposed in the literature to quantify the dexterity of robot manipulators. In this work, we propose the most used one, which is the condition number $\kappa(\mathbf{J})$ of the Jacobean matrix that describes the overall kinematic behavior of a robot [15]. The problem of non homogeneity of the Jacobean matrix is not encountered in our case since the 3-translational-DOF parallel manipulator has only translation degrees of freedom. The local dexterity is defined as :

$$\kappa(\mathbf{J}) = \|\mathbf{J}\| \cdot \|\mathbf{J}\|^{\mathbf{T}} \tag{18}$$

The Jacobian describes the overall kinematic behavior of the considered robot. We adopted for the representation the inverse of the condition number, $\eta = \frac{1}{\kappa(J)}$, ranging between 0 and 1 (isotropy is reached when $\eta = 1$).

The manipulator under study is in a singular configuration if and only if the set of the three vectors $(\mathbf{B}_1\mathbf{C}_1, \mathbf{B}_2\mathbf{C}_2, \mathbf{B}_3\mathbf{C}_3)$ are linearly dependent [2]. This condition depend only on the value of the geometric parameter, the radius R, which appears in the expression of the unit vector $\mathbf{u}_i$. In order to explore the evolution on the local dexterity for a given design vector and over the manipulator workspace, **Fig. 4** illustrates the distribution of the inverse of the condition number in the (x, y) plane and for a given value of the radius R.

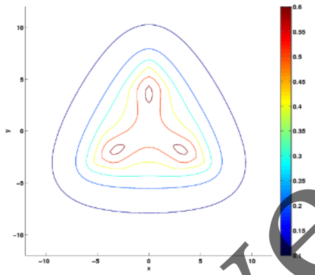

Fig. 4 The local dexterity distribution for $R = 5$ and $z = 5$

5. Synthesis of the RAF robot for a prescribed workspace

5.1Formulation of the problem

The aim of this section is to formulate the multidimensional optimization problem of selecting the design variables for the RAF robot having a specified workspace with the best kinematic performance distribution. The desired workspace is given by a volume $\mathbf{\Omega}$ in space.

The optimization problem can be formulated as follows:

Given : A specified volume in space $\mathbf{\Omega}$.

Find : The parameters of the RAF robot having the smallest workspace that includes the specified volume and best kinematic performance.

The general associated optimization problem, with n parameters for a suitably chosen objective function $F(\mathbf{I}, P)$,can be stated as:

$$\min F(\mathbf{I}, P) = [f_1 \quad f_2]^{\mathrm{T}} \tag{19}$$

Subject to,

$ha_i^{\max}(I, P_k) \leq 0, i = 1,..,3;\ k = 1,..,N_{pt}$: for active workspace constraints.

$ha_i^{\min}(I, P_k) \geq 0, i = 1,..,3;\ k = 1,..,N_{pt}$: for active workspace constraints.

$hp_j(I, P_k) \leq 0, j = 1,2;\ k = 1,..,Npt$:for passive workspace constraints.

For all the points P inside the specified workspace $\mathbf{\Omega}$

where $\mathbf{I} = [x_1, x_2, ..., x_n]$ is the unknown vector of parameters, and $x_i \in [x_{l\min}, x_{l\max}], i = 1,2,..,n$ specify the allowable parameters range for each variable.

In this work, we will take the case where $\mathbf{\Omega}$ is a cube given by $N_{pt} = 8$ points (see **Fig. 5**).

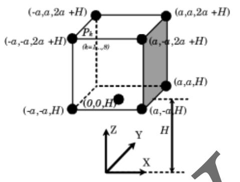

Fig. 5 The scheme of the prescribed workspace

For every workspace to be generated by the RAF robot, the independent design variables are:

$$\mathbf{I} = [r, l_{max}, L, R, H] \tag{20}$$

where, $r = r_A - r_D$: The difference in radius of the passive kinematic chain. $R = r_B - r_C$: The difference in radius of the active kinematic chain. l_{max} : The maximum length of the actuator. L : The length of the leg. H : is a parameter defining how far is the specified volume from the base of the RAF robot. The center of the cube is taken on the z axis because of the symetry of the workspace.

5.1.1 Power function ratio

In a previous work [8], the performed optimization proved that one of the passive workspace or the active workspace can have a great influence on the quality of the optimal solution. This formulation ensures that the desired workspace is obtained but leads to a cumbersome structure. A large difference between the dimensions of the two chains, passive and active, should be noted. Indeed, the two obtained design vectors for the RAF robot present a large base or a large platform. The quality of the obtained results depends on the choice of the value of the aggregation coefficient used in the definition of the objective function. In order to overcome this formulation problem and to obtain the passive and active workspaces with similar sizes, a new formulation based on the use of power function ratio, is proposed. This ratio is defined as:

$$\frac{f_p}{f_a} \simeq 1$$

The corresponding objective function is defined as follows:

$$f_1(\mathbf{I}, P_k) = \left|\frac{f_p}{f_a} - 1\right|$$

where,

$$f_a = \frac{\sum_{i=1}^{3}\left|ha_i^{\max}(\mathbf{I},P_k)\right|}{\sqrt{\sum_{i=1}^{3}\left(ha_i^{\max}(\mathbf{I},P_k)\right)^2}} + \frac{\sum_{i=1}^{3}\left|ha_i^{\min}(\mathbf{I},P_k)\right|}{\sqrt{\sum_{i=1}^{3}\left(ha_i^{\min}(\mathbf{I},P_k)\right)^2}} \text{ and } f_p = \frac{\sum_{j=1}^{2}\left|hp_j(\mathbf{I},P_k)\right|}{\sqrt{\sum_{j=1}^{2}\left(hp_j(\mathbf{I},P_k)\right)^2}}$$

5.1.2 Dexterity

Several methods and dexterity indices can be found in the literature, e.g., Yoshikawa [1], Angeles [2], and Gosselin [3]. To compute the kinematic performance of a structure, we chose the global dexterity method proposed by Gosselin as it characterizes the isotropy of the robot. A commonly used criterion to evaluate this kinematic performance is the global conditioning index η^G, which describes the isotropy of the kinematic performance. The index, for a given structure described by the design vector **I**, is defined over a workspace $\mathbf{\Omega}$ as:

$$\eta^{\mathrm{G}} = \frac{\int_\Omega \eta^{\mathrm{L}} dw}{\int_\Omega dw} = \frac{\int_\Omega 1/\kappa(\mathbf{J}) dw}{\int_\Omega dw} \tag{21}$$

Where η^{L} is the local dexterity and $\kappa(\mathbf{J})$ is the condition number of the kinematic Jacobian matrix (19). The corresponding objective function is defined as follows: $f_2(\mathbf{I}) = \eta^{\mathrm{G}}$

5.2 Results

The objective is to find the smallest set of parameters, given by $\mathbf{I}^*$, that can yield a RAF robot having a workspace with smallest passive/active workspace that includes the given volume in space $\mathbf{\Omega}$, while, simultaneously, achieving the best kinematic performances over the whole workspace. The methodology followed here to solve this problem is based on minimizing the multiple design objectives. This minimization problem is solved using the the Multi-Objective Genetic Algorithm (MOGA) method. The solutions are called Pareto-optimal solutions when an improvement in one objective requires a degradation of another. **Fig. 6** shows the surface representing the Pareto front. Each point represents the values of the two objective functions, respectively f_1 and f_2, obtained by a given design vector.

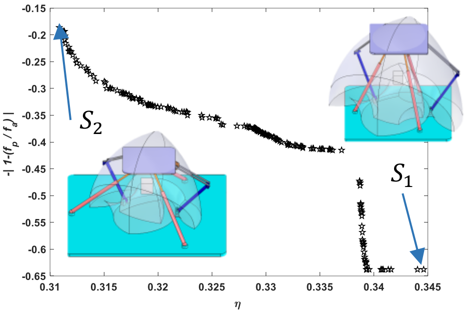

Fig. 6 Pareto front

6 Conclusions

In this work, the workspace of the RAF parallel manipulator having three linear actuators, was determined. An optimal dimensional synthesis method suited for the RAF robot was presented and solved. In this approach, two objective functions were considered. The first one aims at finding the smallest robot having a desired workspace and the second one is to ensure the best overall dexterity over this workspace. The first objective function is based on the concept of the power of a point, which was used to calculate the ratio of the passive to the active workspaces. The optimum value of the ratio is unity, which ensures the two workspaces having similar sizes. The second objective function is based on the condition number of the jacobian matrix. The MOGA method was used to find the optimal solutions represented by the Pareto front. Two extreme solutions from the Pareto front were taken and their CAD models were presented.

It was shown that favoring the dexterity could lead to a bulky robot and a robot with similar workspaces could have a relatively low dexterity. However, the presented solutions all have a value of dexterity ranging from 0.3 to 0.35, which is relatively low.

References

1. Romdhane L., 1999, "Design and analysis of a hybrid serial-parallel manipulator". Mechanism and Machine Theory, Vol. 34, Issue 7, pp 1037-1055.

2. Romdhane L., Affi Z. and Fayet M., 2002, Design and singularity analysis of a 3 translational-DOF in-parallel manipulator, ASME Journal of Mechanical Design, Vol. 124, pp 419-426.
3. Affi Z., Romdhane L. and Maalej A., 2004, Dimensional synthesis of a 3-translational-DOF in-parallel manipulator for a desired workspace, Euro. J. of Mech. - A/Solids, V. 23, I. 2, pp 311-324.
4. M. Ceccarelli, G. Carbone, E. Ottaviano, 2005, An Optimization Problem Approach For Designing Both Serial And Parallel Manipulators. The Int. Sym. on Multibody Systems and Mechatronics Uberlandia, Brazil, 6-9 March
5. Clavel, R., 1988. Delta, a fast robot with parallel geometry. In: Proc. of the 18 the Int. Symp. of Robotic Manipulators. IFR Publication, pp.91-100.
6. Clavel R.,1986, Une nouvelle structure de manipulation parallèle pour la robotique légère, R.A.I.R.O. APII, Vol 23, 6.
7. Lallemand J.P. ,Goudali A. and Zeghloul S., 1997, The 6 - D.o.f. 2 - Delta parallel robot, Robotica Journal, Vol. 15, pp 407-416.
8. M. A. Laribi, A. Mlika, L. Romdhane, S. Zeghloul «Multi criteria optimum design of 3 dof translational parallel manipulators (3TPM)» 13th IFToMM World Congress, Guanajuato, Mexico, June 19-23, 2011,
9. Laribi M.A. , Romdhane L. and Zeghloul S. , 2007, Analysis and dimensional synthesis of the DELTA robot for a prescribed workspace, in Mech. Mach. Th. 42, I. 7, July 2007, pp 859-870.
10. Gosselin C. ,1990, Determination of the workspace of 6-dof parallel manipulators, ASME Journal of Mechanical Design, Vol. 112, pp. 331-336.
11. Boudreau R. and Gosselin C. M.,2001, Mech. Mach. Theory 36, pp 327-342.
12. Boudreau R. and Gosselin C. M. ,1999, ASME J. Mech. Design, Vol 121, pp 533-537.
13. Gallant M. and Boudreau R.,2002 , The synthesis of planar parammel manipulators with prismatic joints for an optimal singularity-free workspace, J. of Robotic Systems 19 (1), pp 13-24.
14. Snyman J. A. and Hay A. M.,2005, Optimal synthesis for a continuous prescribed dexterity interval of 3-DOF parallel planar manipulator for different prescribed output workspaces, Proceeding of CK2005, Cassino May 4-6.
15. Yang, G., et al., 1999. Design, and kinematic an analysis of modular reconfigurable parallel robots. In: Proc. IEEE. ICRA, Michigan.
16. L.-W. Tsai, previous Robot Analysis: The Mechanics of Serial and previous Parallel Manipulator, John Wiley & Sons, Inc. (1999).
17. D. Chablat and P. Wenger, Architecture optimization of a 3-DOF previous parallel mechanism for machining applications: The orthoglide, IEEE Tran. on Rob. and Aut. 19-3, 2003, pp. 403-410.
18. X.-J. Liu, J. Jeong and J. Kim, A three translational DoFs previous parallel cube-manipulator, Robotica 21 (6) (2003), pp. 645-653.
19. Prashant Kumar Jamwal, Shengquan Xie and Kean C. Aw, Kinematic design optimization of a parallel ankle rehabilitation previous robot using modified genetic algorithm , Robotics and Autonomous Systems Volume 57, Issue 10, 31 October 2009, Pages 1018-1027.